ET 570 – Digital Communications

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Session 6 Linear receivers

Transmitted Signal:

$$\underbrace{V(t)cos\omega_c t}_{In-Phase} - \underbrace{W(t)sin\omega_c t}_{Quadrature}$$

Input: $X(t)$+Transmitted Signal ($X(t)$ is the Gaussian Noise.
In-Phase: Multiply by $2cos\omega_c t$ then pass through an LPF $\Longrightarrow$ $X_c(t) + V(t)$
Quadrature: Multiply by $-2sin\omega_c t$ then pass through an LPF $\Longrightarrow$ $X_s(t) + W(t)$

$$H(\omega) = \begin{cases} 1, & |\omega| \leq \omega_c \\ 0, & elsewhere \end{cases}$$

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- Equivalent Noise BW $W_n = \frac{1}{|H_{max}|^2} \int_0^\infty |H(f)|^2 df$

•rms BW $W_{rms}^2 = \frac{\int_{-\infty}^{\infty} f^2 |H(f)|^2 df}{\int_{-\infty}^{\infty} |H(f)|^2 df}$

•Power Transfer Function must fall faster than $\frac{1}{f^2}$.

- Half-Power (3-dB) BW $W_{\frac{1}{2}}$: LPF frequency at which magnitude of power transfer function falls to $\frac{1}{2}$ that at origin.

$$S_Y(f) = S_X(f)|H(f)|^2$$

•BPF width of frequency span between $\frac{1}{2}$ amplitude points of power transfer function around center frequency of passband.

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Now consider our standard receiver structure:

- Let $s_k(t)$ be the transmitted signal, $k \in \{0, 1\}$
- Let $X(t)$ be Gaussian noise, WSS, aero-mean, $S_X(\omega)$
- Let $s_i(t)$ be the signal with noise
- Let $Z_k(t)$ be the signal after $h(t)$
- $s_0(t)$, $s_1(t)$ arbitrary: $0 \rightarrow s_0(t)$ and $1 \rightarrow s_1(t)$

Assume $(s_0 * h)(T_0) > (s_0 * h)(T_0)$; WLOG:

$(s_i * h)(T_0) = \int_{-\infty}^{\infty} s_i(T_0 - \alpha) h(\alpha) d\alpha$

$Z_k(t) = \int_{-\infty}^{\infty} h(t - \alpha)[s_k(\alpha) + X(\alpha)] d\alpha$

$= \int_{-\infty}^{\infty} h(t - \alpha) s_k(\alpha) d\alpha + \int_{-\infty}^{\infty} h(t - \alpha) X(\alpha) d\alpha$

$= \hat{s}_k(t) + \hat{X}(t)$

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Let's look at the estimated output mean:
$\hat{\mu}_k(T_0) \triangleq E\{Z_k(T_0\} = \hat{s}_k(T_0) \quad E\{X(t)\} = 0$, WSS process.
The estimated variance (noise power) is:
$\hat{\sigma}_k^2 \triangleq var\{Z_k(T_0\} = C_{Z_k}(0) = E\{[\hat{X}(T_0)]^2\}$
$= R_{\hat{X}}(0) = E\{[\hat{X}(T_0)]^2\} \quad \forall t.$
Because $Z_k(t)$ is co-variance stationary.

1. $R_{\hat{X}}(0) = \overbrace{(h * \tilde{h} * R_X)}^{R_{\hat{X}}(0)}(0)$
2. $E\{[\hat{X}(T_0)]^2\} = E\{[\int_{-\infty}^{\infty} h(T_0 - \tau)X(\tau)d\tau]^2\}$
   $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(T_0 - \tau)h(T_0 - \lambda)\underbrace{R_X(\tau - \lambda)}_{E\{X(\tau)X(\lambda)\}}d\tau d\lambda$
   $= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(u)h(v)R_X(v - u)dudv$
3. $R_{\hat{X}}(0) = E\{[\hat{X}(t)]^2\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 S_X(\omega)d\omega$

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$Z_k(T_0)$ is therefore a Guassian r.v. with mean $\hat{\mu}_k(T_0)$ and variance of $\hat{\sigma}_k^2$.

$P(\text{error}|0\text{sent}) = p(Z_k(T_0) < \gamma|0\text{sent})$
$= p(Z_0(T_0) < \gamma)$
$= \Phi(\frac{\gamma - \hat{\mu}_0(T_0)}{\hat{\sigma}_0})$
$= \Phi(\frac{\gamma - \hat{S}_0(T_0)}{\sqrt{R_{\hat{X}}(0)}}). \qquad (1)$

$P_{e,1} = p(Z_1(T_0) \geq \gamma)$
$= 1 - p(Z_1(T_0) < \gamma)$
$= 1 - \Phi(\frac{\gamma - \hat{\mu}_1(T_0)}{\hat{\sigma}_1})$
$= \Phi(\frac{\gamma - \hat{S}_1(T_0)}{\sqrt{R_{\hat{X}}(0)}}). \qquad (2)$

$P_{e,0} = 1 - \Phi(\frac{\hat{S}_0(T_0) - \gamma}{\sqrt{R_{\hat{X}}(0)}}). \qquad (1)$