ET 570 – Digital Communications

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Session 5 Random Processes (Continued)

Independent Random Processes:
$X(t)$ and $Y(t)$are independent r.p.'s if $\forall n$; $\forall t_1, t_2, \cdots, t_n$; and $\forall s_1, s_2, \cdots, s_n$;
the random vectors $[X(t_1), X(t_1), \cdots, X(t_n)]$ and $[Y(s_1), Y(s_1), \cdots, Y(s_n)]$ are independent.
i.e., $X_i = X(t_i)$; $\underline{X} = [X_1, X_2, \cdots, X_n]$
$Y_j = Y(s_j)$; $\underline{Y} = [Y_1, Y_2, \cdots, Y_n]$
$X(t)$ and $Y(t)$ are independent r.p.'s iff:
$\forall n$; $\forall t_1, t_2, \cdots, t_n$; and $\forall s_1, s_2, \cdots, s_n$;
$F_{\underline{X},\underline{Y}}(x, y) = F_{\underline{X}}(\underline{x}) F_{\underline{Y}}(\underline{y}), \quad \forall \underline{x}, \underline{y}$.

Session 5 Random Processes (Continued)

FACTS:

1. Independent r.p.'s are uncorrelated.
2. If $X(t)$ and $Y(t)$ are jointly Gaussian and uncorrelated, the they are independent.

Definition: $X(t)$ and $Y(t)$ are jointly WSS r.p.'s if $X(t)$ and $Y(t)$ are each WSS and:

$$R_{X,Y}(t,s) = R_{X,Y}(0, s-t)$$

$\forall t$; $\forall s$; $s = t + \tau$, $s - t = \tau$, $R_{X,Y}(0,\tau)$.

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Session 5 Random Processes (Continued)

Representation of a Narrowband Random Process:
Suppose that $X(t)$ is a WSS r.p. with zero-mean and spectral density $S_X(\omega)$ such that $S_X(\omega) = 0$ for $|\omega| \geq 2\omega_c$.
It may be that $S_X(\omega) \approx 0$ for $\omega \notin (\omega_c - \omega_\beta, \omega_c + \omega_\beta)$ where $\omega_\beta \ll \omega_c$ but this is not required.
Under this condition, it is possible to express $X(t)$ as:

$$X(t) = X_c(t) cos\omega_C t - X_s(t) sin\omega_C t$$

Furthermore, if $X(t)$ is Gaussian, then $X_c(t)$ and $X_s(t)$ are Gaussian.

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CLAIM:

1. $E\{X_c(t)X_c(t+\tau)\} = R_{X_c}(\tau) = R_{X_s}(\tau)$
2. $E\{X_c(t)X_c(t+\tau)\} = R_{X_c,X_s}(\tau) = -R_{X_c,X_s}(\tau)$
3. $E\{X(t)X(t+\tau)\} = R_X(\tau) =$
   $R_{X_c}(\tau)cos\omega_C\tau - R_{X_c,X_s}(\tau)sin\omega_C\tau$
4. $S_x(\omega) = \frac{1}{2}[S_{X_c}(\omega+\omega_C) + S_{X_c}(\omega-\omega_C)] + \frac{j}{2}[S_{X_cX_s}(\omega+\omega_C) +$
   $S_{X_cX_s}(\omega-\omega_C)]$
5. $S_{X_c}(\omega) = Lp[S_{X_c}(\omega+\omega_C) + S_{X_c}(\omega-\omega_C)]$
6. $S_{X_cX_s}(\omega) = Lp[S_{X_c}(\omega+\omega_C) + S_{X_c}(\omega-\omega_C)]$

by Lp[.] is meant the part that is non-zero only for $|\omega| \leq \omega_c$.
PROOFs???

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