ET 570 – Digital Communications

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Session 4 Random Processes

In this lecture, we will introduce/review some basic concepts of Random Processes (RP):

- Correlation Functions
- Gaussian Processes
- Stationary Processes
- Narrow band Noise

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We define mean, autocorrelation, and autocovariance functions for RPs that satisfy $E\{[X(t]^2\} < \infty \quad \forall t$ (2^{nd} order processes):

Mean:
$\mu_X(t) \triangleq E\{X(t)\}$

Autocorrelation Function:
$R_X(t, s) \triangleq E\{(X)t)X(s)\}$

Autocovariance Function:
$C_X(t, s) \triangleq E\{[(X)t) - \mu_X(t)][X(s) - \mu_X(s)]\}$

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Definition: A process $X(t)$ is Wide Sense Stationary (WSS) if:

1. $\mu_X(t) = \mu_X(0) \quad \forall t$
2. $R_X(t, t+\tau) = R_X(0, \tau) = R_X(\tau)$
3. If $C_X(t, t+\tau) = C_X(0, \tau) = C_X(\tau)$

Then $X(t)$ is covariance stationary.
Example: If $X(t) =$ a WSS Gaussian Process
$s(t) =$ a deterministic signal, not constant
$Y(t) = X(t) + s(t)$
Is $Y(t)$ WSS? NO!!!
Is $Y(t)$ covariance stationary? YES!!! [3-48]

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$C_X(t,s) = R_X(t,s) - \mu_X(t)\mu_X(s)$
$Cov(X,Y) = E\{[X - E\{X\}][Y - E\{Y\}]\} = E\{XY\} - E\{X\}E\{Y\}$
$\therefore 1) + 2) \longrightarrow 3)$
But reverse is not true
WSS $\longrightarrow$ covariance stationary
WSS $\longrightarrow \mu_X . R_X(\tau), C_X(\tau)$

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Definition: Cross-correlation function of $X(t)$ and $Y(t)$:
$R_{X,Y}(t,s) \triangleq E\{X(t)Y(s)\}$

$C_{X,Y}(t,s) \triangleq E\{[X(t)-\mu_X(t)][Y(s)-\mu_Y(s)]\} \quad \forall s, \quad \forall t.$
If $R_{X,Y}(t,s) = \mu_X(t)\mu_Y(s), \quad \forall t, s$
the RPs $X(t)$ and $Y(t)$ are uncorrelated.
$X(t)$ and $Y(t)$ are uncorrelated RPs iff
$C_{X,Y}(t,s) = 0 \quad \forall t, s$
$C_{X,Y}(t,s) = R_{X,Y}(t,s) - \mu_X(t)\mu_Y(s)$ [3-57].

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