ET 570 – Digital Communications

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Session 3 erfc (x) and Q(x)

Consider a Gaussian r.v. with mean μ and variance σ^2.
Standard Normal: $\mu = 0$, $\sigma^2 = 1$. The probability distribution function (p.d.f.) is:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

For arbitrary μ and σ,

$$f_{\mathbb{X}}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sigma}\Phi(\frac{x-\mu}{\sigma})$$

Now let's consider our standard receiver system:

$h(t) = P_T(t)$, Threshold device operating at $t = nT$, where n is an integer.

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Denote the signal after the filter as $y(t)$, then:

$$y(t) = \pm T + X_{noise}$$

Here X_{noise} is a Gaussian r.v. with zero-mean. What is the variance?
Answer: $\frac{N_0 T}{2}$. (Give the AWGN spectral density of $\frac{N_0}{2}$.)
Why: Let $f = h * \tilde{h} = T - |t|$, for $|t| \leq T$ (zero otherwise),
$R_y(\tau) = \frac{N_0}{2}\delta(\tau) * f = \frac{N_0}{2} f(\tau)$.
When sampled at $t = nT$, the noise power is $\frac{N_0 T}{2}$.

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Def. An Error Function is defined as:

$$erf(x) \triangleq \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

Def. A Complementary Error Function is defined as:

$$erfc(x) \triangleq 1 - erf(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$$

Def. A Q-function is defined as:

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt \qquad x \geq 0$$

NOTE:

$$Q(x) = \frac{1}{2} erfc(\frac{x}{\sqrt{2}})$$

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What is the density function of $y(t)$ look like given that a "0" was sent?

Given that 0 is sent, distribution function is:

$$\Phi(\frac{x-\mu}{\sigma}) = \Phi(\frac{x-T}{\sigma})$$

$P_{e,0} = Pr\{y(t) < 0|0 sent\}$

$= \Phi(\frac{-T}{\sqrt{N_0 T/2}}) = \Phi(-\sqrt{\frac{2T}{N_0}})$

$= Q(\sqrt{\frac{2T}{N_0}}) = Q(\sqrt{\frac{2\epsilon_b}{N_0}})$

Where ϵ_b is the bit energy. In this case, transmitted energy per bit is $\epsilon_b = T$. (WHY???)

What is $P_{e,1}$???

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Now define Signal-to-Noise Ratio (SNR)as:

$$SNR \triangleq \frac{Desired \quad Signal \quad Component}{\sqrt{Variance \quad of \quad Noise}}$$

Then:

$$P_{e,0} = Q(SNR) = P_{e,1}$$

RECEIVER PARAMETERS:

$$h(t), \qquad T_0, \qquad \gamma$$

(look at a more general system, T_0 is the time when $y(t)$ is sampled.)