ET 570 – Digital Communications

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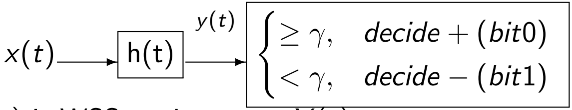
Session 2 Random Variables and Processes

Consider a Linear System below:



Input $X(t)$ is WSS, so is output $Y(t)$.
Define:

$$\tilde{h}(t) = h(-t)$$

Then:

$$\mu_y = \mu_x * h$$

$$R_y(\tau) = R_x(\tau) * h * \tilde{h}$$

$$\begin{aligned}
R_y(\tau) &= E\{Y(t)Y(t+\tau)\} \\
&= E\{\int_{-\infty}^{\infty} X(t-\alpha)h(\alpha)d\alpha \int_{-\infty}^{\infty} X(t+\tau-\beta)h(\beta)d\beta\} \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\{X(t-\alpha)X(t+\tau-\beta)\}h(\alpha)h(\beta)d\alpha d\beta \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_x(\tau+\alpha-\beta)h(\alpha)h(\beta)d\alpha d\beta \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_x(\tau-\gamma-\beta)\tilde{h}(\gamma)h(\beta)d\gamma d\beta \qquad (let \gamma = -\alpha)
\end{aligned}$$

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Spectral density for the W.S.S. Random Process $X(t)$ is:

$$S_x(\omega) = \mathfrak{F}\{R_x\} = \int_{-\infty}^{\infty} R_x(\tau) e^{-j\omega\tau} d\tau$$

$R_y = \tilde{h} * h * R_x \Longleftrightarrow S_y(\omega) = |H(\omega)|^2 S_x(\omega)$

<u>Thermal Noise</u>

Represented by Additive White Gaussian Noise (AWGN) process.

$$S_x(\omega) = \frac{N_0}{2} \qquad \forall\omega$$

$$\mathfrak{F}^{-1}\{S_x(\omega)\} = R_x(\tau)$$

$$\mathfrak{F}^{-1}\{\frac{N_0}{2}\} = \frac{N_0}{2}\delta(\tau)$$

Also, $\int_{-\infty}^{\infty} g(\alpha) X(\alpha) d\alpha$ is a Gaussian (normal) random variable where $g(\alpha)$ is a known function.