ET 570 Digital Communications
Homework Assignment 1

Due: February 10, 2010

January 31, 2010

1. A stationary white noise process with a spectral density of $\frac{N_0}{2} = 0.02V^2/Hz$ is passed through an RC low-pass filter with a time constant of $2ms$. Calculate the rms voltage of the filter output.

2. Find the half-power and rms bandwidths of the random process with the following auto-correlation function:

$$R(\tau) = \begin{cases} 1 - |10^3\tau|, & if|\tau| \leq 10^{-3} \\ 0, & otherwise \end{cases}$$

3. Calculate the variance of the output of a finite time integrator, whose impulse response is given by $h(t) = rect(\frac{t}{T})$, when the input is a stationary white noise with a one-sided spectral density of N_0.

4. Find the half-power and equivalent noise bandwidths of the finite time integrator in Problem 3.

NOTES

1. For a low-pass filter, the half-power bandwidth corresponds to that frequency at which the magnitude of the power transfer function falls to one half of its value at the origin.

2. For a system with transfer function H(f), the equivalent noise bandwidth is defined as:

$$\text{Equivalent Noise BW} = \frac{1}{|H_{max}|^2} \int_0^\infty |H(f)|^2 df$$

where H_{max} is the maximum value of $|H(f)|$. The equivalent noise bandwidth corresponds to the width of an ideal band-pass filter with a gain equal to the maximum gain of the system, and which outputs the same average power from a stationary white noise input as the actual system does.

3. For a low-pass system with transfer function H(f), the rms bandwidth is defined as:

$$\text{RMS BW} = [\frac{\int_{-\infty}^{\infty} f^2|H(f)|^2 df}{\int_{-\infty}^{\infty} |H(f)|^2 df}]^{\frac{1}{2}}$$

This bandwidth measures the radius of gyration of $|H(f)|$ around the origin.