# ET 644 – Advanced Digital Signal Processing

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# Session 9 DAC

Outline:

- Sample and Hold (S & H), Reconstruction - section 9.3.1
- Digital Up-Sampling (Section 10.3)
- Efficient Up-sampling (Section 10.5.2)
- Reminder: Exam 1 covers up to this class.

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- See Fig. 9.22 on p.767 $\Longrightarrow$ S/H reconstruction
- Digital Up-Sampling (Increasing sampling rate just prior to D/A conversion)

$$x[n] = x_a(\tfrac{n}{F_s}) \longrightarrow \boxed{\text{DT System}} \longrightarrow y[n] = x_a(\tfrac{n}{LF_s})$$

- where L = integer
- i.e., increasing sampling rate by "L" just prior to S/H.
- e.g., A CD player advertised "8 times over sampling" meaning L = 8.

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- Mathematical Preamble:

$$x[n] * \delta[n - n_0] = x[n - n + 0]$$

- In particular, if $n_0 = 0$

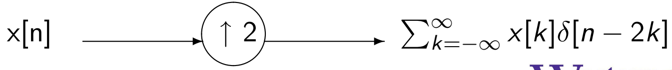
$$x[n] = \Sigma_{k=-\infty}^{\infty} x[k]\delta[n - k]$$

- Consider: $\sum_{k=-\infty}^{\infty} x[k]\delta[n - 2k]$

$= \cdots + x[-1]\delta[n+2] + x[0]\delta[n] + x[1]\delta[n-2] + \cdots$

$= \{\cdots, 0, x[-1], 0, x[0], 0, x[1], 0, \cdots\}$

- See graphical representation of the system.



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- More generally:
- Insert L-1 zeros between each pair of samples


- End of Preamble.
- As long as $F_s > 2W$, (i.e., $|X_a(F) \approx 0, \text{for}|F| > W$)
- Then $x_a(t)$ maybe reconstructed as:

$x_a(t) = \sum_{k=-\infty}^{\infty} x[k]h_{LP}(t - kT_s)$

- See graphical presentation.

$y[n] = x_a(n\frac{T_s}{L}) = x_a(\frac{n}{LF_s})$

$= \sum_{k=-\infty}^{\infty} x[k]h_{LP}\underbrace{(n\frac{T_s}{L} - kT_s)}_{(n-kL)\frac{T_s}{L}}$

$= \sum x[k]h_{LP}[n - kL]$

where $h_{LP}[n] = h_{LP}(n\frac{T_s}{L}) = h_{LP}(\frac{n}{LF_s})$.

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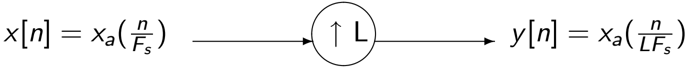
$h_{LP}[n-kL] = h_{LP}[n] * \delta[n-kL]$
$y[n] = \{\sum_{k=-\infty}^{\infty} x[k]\delta[n-kL]\} * h_{LP}[n]$

- Convolution is distributive




- Now let's investigate the specs of this low-pass filter.
- Example: for speech with bandwidth $\approx$ 4kHz, sampled at $F_s = 12kHz$.
- Desire to increase sampling rate by L=2 to 24 kHz.
- $\omega_p = 2\pi\frac{4}{2(12)} = \frac{\pi}{3}$
- $\omega_s = 2\pi\frac{12-4}{2(12)} = \frac{2\pi}{3}$
- See demo upsamplex2eg1.m

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- Efficient Digital Upsampling
- L = 2 for illustration

$y[n] = \sum_{k=-\infty}^{\infty} x[k] h_{LP}[n-2k]$

- Consider y[2n] and y[2n+1] separately.

$y_0[n] = y[2n] = \sum_{k=-\infty}^{\infty} x[k] h_{LP}[\underbrace{2n-2k}_{2(n-k)}] = x[n] * h_0[n]$

•where: $h_0[n] = h_{LP}[2n]$

$x[n] \longrightarrow \boxed{h_0[n] = h_{LP}[2n]} \longrightarrow y_0[n] = y[2n]$

- Similarly,

$y_1[n] = y[1+2n] = \sum_{k=-\infty}^{\infty} x[k] h_{LP}[\underbrace{1+2n-2k}_{1+2(n-k)}] = x[n] * h_1[n]$

•where: $h_1[n] = h_{LP}[1+2n]$

- See graphical presentation of the "commutator." (see demo: upsamplex2eg2.m)

Western Carolina UNIVERSITY