ET 644 – Advanced Digital Signal Processing

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Session 8 Sampling Theorem

Outline:

- Aliasing Examples
- Sampling Theorem - Ideal Reconstruction Formula (Section 4.2.9)
- DAC - Digital to Analog Conversion (Section 9.3)

Session 8 Sampling Theorem

- Derive Ideal Reconstruction Formula
- If $F_s > 2W$, then

$X(\omega) = F_s X_a(\frac{F_s}{2\pi}\omega) \qquad$ for $|\omega| < \pi$.

- In turn:

$X_a(F) = \frac{1}{F_s} X(\frac{2\pi F}{F_s}), \qquad \omega = \frac{2\pi F}{F_s}, \quad$ for $-\frac{F_s}{2} < F < \frac{F_s}{2}$

$= \frac{1}{F_s} \Sigma_{n=-\infty}^{\infty} x[n] e^{-j\frac{2\pi F}{F_s}n} \qquad$ for $|F| < \frac{F_s}{2}$

- Taking IFT of $X_a(F)$ gives:

$x_a(t) = \int_{-\infty}^{\infty} X_a(F) e^{+j2\pi Ft} dF$

$x_a(t) = \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} \frac{1}{F_s} \Sigma_{n=-\infty}^{\infty} x[n] e^{-j2\pi\frac{F}{F_s}n} e^{+j2\pi Ft} dF$

- Working out the integral, we get:

$$x_a(t) = \Sigma x[n] \frac{sin[\frac{\pi}{T_s}(t - nT_s)]}{\frac{\pi}{T_s}(t - nT_s)}$$

See pp. 274-275.

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- Diversion: Dirac Delta Function

$$\delta_a(t) = \lim_{\Delta \to 0} \frac{1}{\Delta} rect(\frac{t}{\Delta})$$

- Some properties:
  - $\delta_a(t) = 0, \text{for} t \neq 0;$
  - $\int_{t_0-\epsilon}^{t_0+\epsilon} \delta_a(t - t_0)dt = 1$
- Sifting property:

$$\int_{-\infty}^{\infty} x_a(t)\delta_a(t - t_0)dt = x_a(t_0)$$

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- Convolution property:

$$x_a(t) * \delta_a(t - t_0) = \int_{-\infty}^{\infty} x_a(t - \lambda)\delta_a(\lambda - t_0)d\lambda = x_a(t - t0)$$

- $\delta_a(t - t_0) \overset{CTFT}{\longleftrightarrow} ?$
$\int_{-\infty}^{\infty} \delta_a(t - t_0)e^{-j2\pi FT} dt$
$= e^{-j2\pi F t_0}, \; -\infty < F < \infty$
End of Diversion.

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$$x_a(t) = \Sigma_{n=-\infty}^{\infty} x[n] \underbrace{\frac{sin[\frac{\pi}{T_s}(t-nT_s)]}{\frac{\pi}{T_s}(t-nT_s)}}_{=\frac{sin(\frac{\pi}{T_s}t)}{\frac{\pi}{T_s}t} * \delta_a(t-nT_s)}$$

$$x_a(t) = \{\Sigma_n x_a(nT_s)\delta_a(t-nT_s)\} * \frac{sin(\frac{\pi}{T_s}t)}{\frac{\pi}{T_s}t}$$

- $h_{LP}(t) = \frac{sin(\frac{\pi}{T_s}t)}{\frac{\pi}{T_s}t}$ - Impulse response of Ideal Lowpass Filter (ILPF). What is the CTFT of the above $h_{LP}(t)$?

$\Sigma_n x_a(nT_s)\delta_a(t-nT_s) = x_s(t) \longrightarrow \boxed{\text{ILPF, cutoff} = \frac{F_s}{2}} \longrightarrow x_a(t)$.

- What is the CTFT of $x_s(t)$? - Derivation in class.