ET 644 – Advanced Digital Signal Processing

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session 6 DT System Analysis

Outline:

- Effects of pole and zero locations on frequency response (Section 4.5.1)
- See zpgui.m (demo)
- Discrete Time Fourier Transform
- Digital Notch Filters - section 4.5.4

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- Frequency Analysis of DT Signals
- Arbitrary (stable) x[n] may be represented as:
- $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega$ - Inverse DTFT
- where $X(\omega)$ is the DTFT of x[n].

$X(\omega) = \Sigma_{n=-\infty}^{\infty} x[n]e^{-j\omega n} = X(z)|_{z=e^{j\omega}}$

- Note: $e^{j(\omega+2\pi)n} = e^{j\omega n}e^{j2\pi n} = e^{j\omega n}$
- Riemann integration dictates:

$x[n] = \lim_{N\to\infty}\{\Sigma_{k=-N}^{N}(\frac{1}{N}X(\frac{k\pi}{N})e^{-j\frac{k\pi}{N}n})\}$

$$\underbrace{\Delta\omega = \frac{2\pi}{N}}_{width} x \underbrace{\frac{1}{2\pi}X(\frac{k\pi}{N})e^{-j\frac{k\pi}{N}n}}_{height}$$

- of N rectangles, equi-spaced over $-\pi < \omega < \pi$.

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• x[n] is an (infinite) sum of (complex) sinewaves infinitesimally spaced in frequency.

• recall $x[n] \overset{h[n]}{\longmapsto} y[n]$
$\Longrightarrow$ Y(z) = H(z)X(z)

• On unit circle: $Y(\omega) = H(\omega)X(\omega)$

• $x[n] * h[n] \overset{DTFT}{\longleftrightarrow} H(\omega)X(\omega)$

•Through judicious positioning of zeros and poles can:

- emphasize "desired" frequency bands
- de-emphasize other frequency bands

• See Fig. 4.43 on p. 331 and Fig. 4.44 on p.334.

• Example: $y[n] = -\frac{1}{2}y[n-2] + x[n] + x[n-1]$

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- Notch Filters

$$H_{notch}(z) = \frac{G(z - e^{j\omega_0})(z - e^{-j\omega_0})}{(z - re^{j\omega_0})(z - re^{-j\omega_0})}$$

- where ω_0 is the frequency to be notched.
- Graphical presentation of a notch filter structure.
- See Fig. 4.5.1 and 4.5.2 in text

$$\begin{aligned} y[n] = {} & 2rcos(\omega_0)y[n-1] - r^2y[n-2] \\ & + Gx[n] + G2cos(\omega_0)x[n-1] + Gx[n-2] \end{aligned}$$