ET 644 – Advanced Digital Signal Processing

Fall 2009

Department of Engineering and Technology
zhang@email.wcu.edu

session 5 Z-Transform (Cont'd)

Outline:

- Z-transform continued
- Sections 3.4.3 and 3.6.4
- Relationship between ZT and DTFT
- Sections 4.2.6 and 4.4.2

Western Carolina UNIVERSITY

session 5 Z-Transform (Cont'd)

Outline:

- Example

$y[n] = \frac{13}{4} y[n-1] - \frac{3}{4} y[n-2] + x[n]$

- Determine all possible impulse responses
- Determine ROCs and Draw pole-zero diagram
- Determine if the system is stable and causal for each ROC

session 5 Z-Transform (Cont'd)

- Stability: requires ROC to include unit circle, $|z| = 1$
- Supporting argument:
- Invoke triangle inequality: $|a + b| \leq |a| + |b|$
- $|H(z)| \leq \Sigma_{n=-\infty}^{\infty} |h[n]z^{-n}|$
- On unit circle: $|z^{-n}| = |z|^{-n} = \frac{1}{1^n} = 1$

session 5 Z-Transform (Cont'd)

- Stability and Causality
- Requires $|z| > |p_N|$ must include unit circle, $|z| = 1$

$\Longrightarrow |p_N| < 1$

$\Longrightarrow$ All poles ,ust be located within unit circle

- Note: for distinct poles: $h[n] = \Sigma_{k=1}^{N} A_k p_k^n u[n]$
- On unit circle:: $|H[z]| \leq \underbrace{\Sigma_{n=-\infty}^{\infty} |h[n]| < \infty}_{\text{for BIBO Stability}}$
- ROC must include unit circle for BIBO stability.

session 5 DTFT

- Onto Chapter 4 on DTFT - Discrete-Time Fourier Transform
- Recall: $x[n] = z_0^n \overset{LTI, h[n]}{\longmapsto} y[n] = H(z_0)z_0^n \quad \forall n$
- Consider z_0 on the unit circle
- Recall $e^{j\theta} = cos\theta + jsin\theta$
- $z_0 = e^{j0} = 1; \quad z_0 = e^{j\pi} = -1 = e^{-j\pi}$
- $z_0 = e^{j\frac{pi}{2}} = j; \quad z_0 = e^{-j\frac{pi}{2}} = -j$

- $x[n] = e^{j\omega_0 n} \overset{LTI, h[n]}{\longmapsto} y[n] = H(\omega_0)e^{j\omega_0 n} \quad \forall n$
- where $H(\omega) = H(z)|_{z=e^{j\omega}}$ - Notational Problem ...
- $H(\omega) = \Sigma_{n=-\infty}^{\infty} h[n]e^{j\omega n}$ - DTFT of h[n]
- Only defined if ROC includes $|z| = 1$

$\Longrightarrow$ Only defined for stable systems.