Western Carolina UNIVERSITY

ET 644 – Advanced Digital Signal Processing

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session 3 Correlation & PN Sequence

Outline:

- Correlation of DT Signals
- Relevant sections in P & M Text: Sect. 2.6
- Pseudo-Noise Sequences
- Application to Radar
- Application to Spread Spectrum Communications - see cdmaeg.m

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- Define Cross-Correlation:

$r_{xy}(k) = \Sigma_{n=-\infty}^{\infty} x[n]y[n-k]$
- Auto-Correlation:

$r_{xx}(k) = \Sigma_{n=-\infty}^{\infty} x[n]x[n-k]$

The above definitions are for deterministic cases.
- for each lag k:
  - shift right by k
  - point-wise multipy
  - sum

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- Relative to convolution, missing initial "fold" step:

$r_{xy}(k) = x[k] * y[-k]$

$r_{xx}(k) = x[k] * x[-k]$

- Example: Pseudo-Noise Sequences

$x[n] = \{\underbrace{1}_{n=0}, 1, 1, -1, -1, 1, -1\}$

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- Relative to convolution, missing initial "fold" step:

$r_{xy}(k) = x[k] * y[-k]$

$r_{xx}(k) = x[k] * x[-k]$

- Example: Pseudo-Noise Sequences

$x[n] = \{\underbrace{1}_{n=0}, \quad 1, \quad 1, \quad -1, \quad -1, \quad 1, \quad -1\}$

$k = 0, \qquad \{1, \quad 1, \quad 1, \quad -1, \quad -1, \quad 1, \quad -1\},$

$r_{xx}[0] = 7$

$k = 1, \qquad \{1, \quad 1, \quad 1, \quad -1, \quad -1, \quad 1, \quad -1\},$

$r_{xx}[1] = 0$

$r_{xx}[2] = -1,\ r_{xx}[3] = 0,\ r_{xx}[4] = -1,\ r_{xx}[5] = 0,\ r_{xx}[6] = -1$

$r_{xx}[k] = 0$ for $|k| > 6$, $r_{xx}[k] = r_{xx}[-k] \implies$ even function.

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- Graphical presentation of the above example
  - example of a barker code
  - sharp peak at k = 0
  - time-delay estimation in radar
  - transmit pulse $S_a(t)$
  - see Fig. 2.38 on p. 119

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- Received "echo" after reflection off object
$y_a(t) = \Gamma S_a(t - \tau_d) + W_a(t)$
  - Γ is an unknown amplitude
  - τ_d is the round-trip time-delay
  - $W_a(t)$ is the noise
- Sampled version:
$y[n] = \Gamma S_a(nT_s - \tau_d) + W[n]$
- assume sampling high enough, $tau_d = DT_s$, where "D" is an integer.

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$S_a(nT_s - DT_s) = S_a((n-D)T_s) = S[n-D]$, where
$S[n] = S_a(nT_s)$.

- DT Model:

$y[n] = \Gamma S[n-D] + W[n]$
$\tau_d = DT_s = \frac{2R}{c}$
where R = Range to target, c = speed of light.

- use cross-correlation to estimate D:

$$R = \frac{cDT_s}{2}$$

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Here is how you estimate "D":

$$r_{ys}(k) = \Sigma_n y[n] \underbrace{s[n-k]}_{\text{stored in memory}}$$

$$= \Sigma_{n=-\infty}^{\infty}(\Gamma s[n-D] + w[n])s[n-k]$$

$$= \Gamma\Sigma_{n=-\infty}^{\infty} s[n-D]s[n-k] + \Sigma_{n=-\infty}^{\infty} w[n]s[n-k] \text{ (A)}$$

• Change of variables: $n^{'} = n - D, \longrightarrow n = n^{'} + D$:

$$(A) = \Gamma\Sigma_{n^{'}=-\infty}^{\infty} s[n^{'}]s[n^{'} - (k-D)] + r_{ws}[k]$$

$$= \Gamma r_{ss}[k-D] + r_{ws}[k].$$

NOTE: This peaks at k = D!!!

Graphical Presentation ...

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- Example: a = constant, system initially at rest:

$y[n] = ay[n-1] + x[n]$

- Impulse Response: when $x[n] = \delta[n]$

$h[n] = ah[n-1] + \delta[n]$

$h[n] = 0$, for $n < 0 \longrightarrow$ causal system.

- for n = 0:

$h[0] = a\overbrace{h[-1]}^{=0} + 1 = 1$

for $n > 0$, $h[n] = ah[n-1]$.

Consider: $a^n = aa^{n-1}$

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$h[n] = a^n u[n]$

- input = x[n] = u[n]

output $y[n] = ?$

$y[n] = h[n] * x[n] = \{a^n u[n]\} * u[n] = \Sigma_{k=0}^{n} a^k$

$y[n] = \frac{1-a^{n+1}}{1-a}$

See p.81 in P & M Text.