Session 15

- Example of Designing IIR filters via Bilinear Tranformation

- 2^{nD} order Butterworth LPF with a 3-dB cut-off at $\Omega_c = 1$ has the transfer function:

$$H_a(s) = \frac{1}{s^2 + \sqrt{2}\, s + 1}$$

- Use Bilinear Transform $s = c\,\frac{z-1}{z+1}$ to design a digital filter with 3dB cut-off at $\omega_c = \frac{\pi}{2}$ rads./sec.

(a) $\Omega_c = c\tan(\omega_c/2)$

$$1 = c\tan\left(\tfrac{\pi}{2}/2\right) \Rightarrow c = 1$$

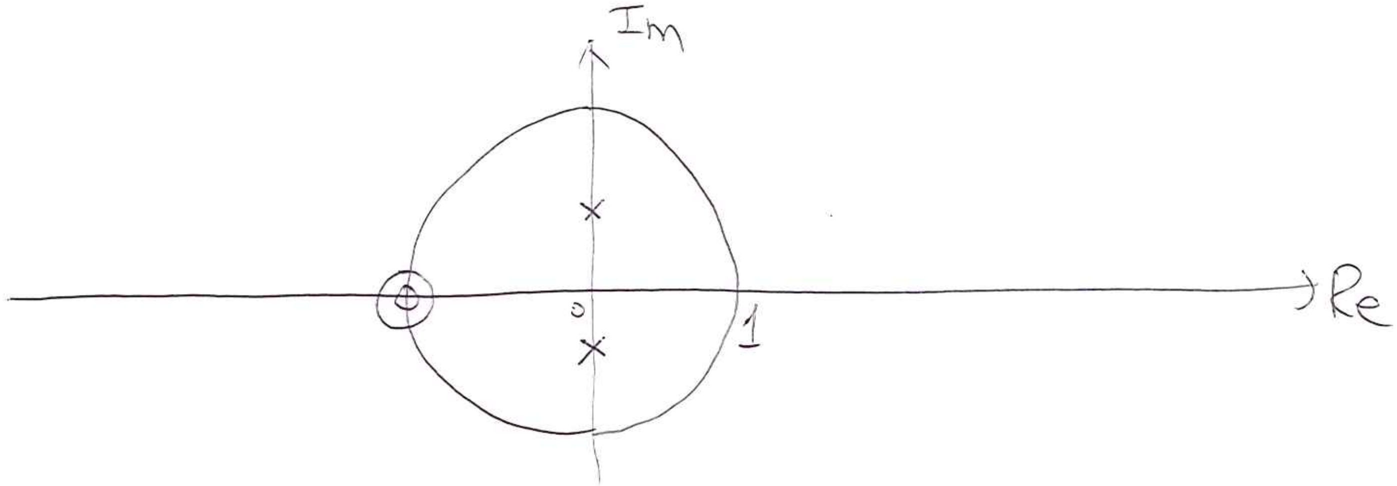
$$H(z) = H_a(s)\Big|_{s = c\frac{z-1}{z+1}}$$

$$= \frac{1}{\frac{(z-1)^2}{(z+1)^2} + \sqrt{2}\,\frac{z-1}{z+1} + 1} \cdot \frac{(z+1)^2}{(z+1)^2}$$

$$= \frac{(z+1)^2}{(z-1)^2 + \sqrt{2}(z^2-1) + (z+1)^2}$$

$$= \frac{(z+1)^2}{(2+\sqrt{2})z^2 + 2-\sqrt{2}}$$

• poles: $p_k = \pm j\sqrt{\dfrac{2-\sqrt{2}}{2+\sqrt{2}}} = \pm j\,0.414$



$$\Rightarrow \quad H(z) = \frac{1 + 2z^{-1} + z^{-2}}{2+\sqrt{2} + (2-\sqrt{2})z^{-2}}$$

$$\Rightarrow \quad y[n] = -\frac{2-\sqrt{2}}{2+\sqrt{2}}\, y[n-2] + \frac{1}{2+\sqrt{2}}\left\{x[n] + 2x[n-1] + x[n-2]\right\}$$