



ET 644 – Advanced Digital Signal Processing

Fall 2009

Department of Engineering and Technology
zhang@email.wcu.edu

Session 12 Downsampling & Subbanding

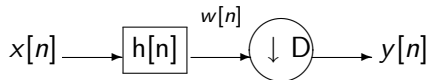
Outline:

- Efficient Downsampling (Section 10.5.2)
- Fractional Sampling Rate Conversion (Section 10.4)
- Digital Subbanding (Section 10.7.2 and 10.9.7)
- Math Preamble:

$$\sum_{k=-\infty}^{\infty} \delta[n-kL] = \sum_{k=0}^{L-1} \frac{1}{L} e^{j2\pi \frac{kn}{L}} = \frac{1}{L} \frac{1 - e^{j2\pi n}}{1 - e^{j2\pi \frac{n}{L}}} = \begin{cases} 1, & n = mL, m \text{ integer} \\ 0, & \text{otherwise} \end{cases}$$

Session 12 Downsampling & Subbanding

- Efficient Downsampling



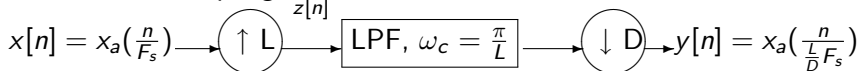
$$\begin{aligned}y[n] &= \sum_{k=-\infty}^{\infty} h[k]x[Dn - k] \\&= \sum_{m=0}^{D-1} \sum_{k'=-\infty}^{\infty} h[k'D + m]x[Dn - (k'D + m)] \\&= \sum_{m=0}^{D-1} \sum_{k'=-\infty}^{\infty} h[kD + m]x[D(n - k) - m]\end{aligned}$$

- See graphical presentation of efficient downsampling structure.

Session 12 Downsampling & Subbanding

- Digital Subbanding
- Preamble: Analog Frequency Division Multiplexing
- Transmit multiple signals simultaneously in different frequency bands
- Follows from either modulation property of CTFT (or just demodulation in time domain) at the receiver end.

- Fractional Sampling Rate Conversion



$$\omega_c = \min\{\frac{\pi}{L}, \frac{\pi}{D}\}$$

Session 12 Downsampling & Subbanding

- Digital Subbanding
- Suppose we sample $x_{1a}(t)$ and $x_{2a}(t)$ at/near Nyquist rate
 $x_1[n] = x_{1a}(\frac{n}{2W})$, $x_2[n] = x_{2a}(\frac{n}{2W})$
- Both signals occupy entire digital frequency band $-\pi < \omega < \pi$
(both periodic with period of 2π .)
- How do we place the signals in different subbands?
- Can't do it by modulation (i.e., multiplying by a cosine)
- Answer: First do digital upsampling to subband the signals
- Then perform digital downsampling to recover the original signals
- See in-class example.