ET 644 – Advanced Digital Signal Processing

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Session 12 Downsampling & Subbanding

Outline:

- Efficient Downsampling (Section 10.5.2)
- Fractional Sampling Rate Conversion (Section 10.4)
- Digital Subbanding (Section 10.7.2 and 10.9.7)

- Math Preamble:

$$\sum_{k=-\infty}^{\infty} \delta[n-kL] = \sum_{k=0}^{L-1} \frac{1}{L} e^{j2\pi \frac{kn}{L}} = \frac{1}{L} \frac{1-e^{j2\pi n}}{1-e^{j2\pi \frac{n}{L}}} = \begin{cases} 1, & n = mL, m \text{integer} \\ 0, & \text{otherwise} \end{cases}$$

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- Efficient Downsampling

$$x[n] \longrightarrow \boxed{\text{h[n]}} \xrightarrow{w[n]} \downarrow \text{D} \longrightarrow y[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[Dn-k]$$
$$= \sum_{m=0}^{D-1} \sum_{k'=-\infty}^{\infty} h[k'D+m]x[Dn-(k'D+m)]$$
$$= \sum_{m=0}^{D-1} \sum_{k'=-\infty}^{\infty} h[kD+m]x[D(n-k)-m)]$$

- See graphical presentation of efficient downsampling structure.

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- Digital Subbanding
- Preamble: Analog Frequency Division Multiplexing
- Transmit multiple signals simultaneously in different frequency bands
- Follows from either modulation property of CTFT (or just demodulation in time domain) at the receiver end.

- Fractional Sampling Rate Conversion

$$x[n] = x_a(\tfrac{n}{F_s}) \longrightarrow \boxed{\uparrow \mathrm{L}} \xrightarrow{z[n]} \boxed{\mathrm{LPF},\ \omega_c = \tfrac{\pi}{L}} \longrightarrow \boxed{\downarrow \mathrm{D}} \longrightarrow y[n] = x_a(\tfrac{n}{\frac{L}{D}F_s})$$

$\omega_c = min\{\frac{\pi}{L}, \frac{\pi}{D}\}$

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- Digital Subbanding
- Suppose we sample $x_{1a}(t)$ and $x_{2a}(t)$ at/near Nyquist rate

$x_1[n] = x_{1a}(\frac{n}{2W})$, $\qquad x_2[n] = x_{2a}(\frac{n}{2W})$

- Both signals occupy entire digital frequency band $-\pi < \omega < \pi$

(both periodic with period of 2π.

- How do we place the signals in different subbands?
- Can't do it by modulation (i.e., multiplying by a cosine)
- Answer: First do digital upsampling to subband the signals
- Then perform digital downsampling to recover the original signals
- See in-class example.