ET 644 – Advanced Digital Signal Processing

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Session 11 Upsampling & Downsampling

Outline:

- Frequency domain analysis of:
- Up-Sampling
- Down-Sampling
- Efficient Down-sampling
- Fractional Sampling Rate Conversion.
- Textbook Sections 10.2, 10.3, 10.9.8

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Session 11 Upsampling & Downsampling

- Recall:

- Passband Edge: $\omega_p = \frac{2\pi W}{LF_s}$
- Stopband Edge: $\omega_s = \frac{2\pi}{L} - \frac{2\pi W}{LF_s}$
- $z[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n - kL]$
- Efficient implementation for general case of upsampling by L: no zeros inserted, but take x[n] and run it through L filters, then interleave the outputs of each filter to obtain the final output.
- See graphical presentation.

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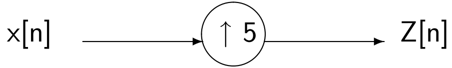
- Frequency domain analysis of upsampling

$Z(\omega) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[k]\delta[n - kL]e^{-j\omega n}$
$= \sum_{k=-\infty}^{\infty} x[k] \sum_{n=-\infty}^{\infty} \delta[n - kL]e^{-j\omega n}$
$= \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega kL} = X(L\omega)$

- Periodic ω, period $= \frac{2\pi}{L}$
- Example: Given $X(\omega)$, what is $Z(\omega)$ if x[n] is applied to the following upsampling system?

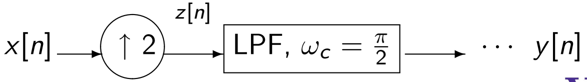


- LPF removes the compressed replicas ("images") lying in $-\pi < \omega < \pi$ except that centered at $\omega = 0$
- Referred to as anti-imaging filter
- if $F_s > 2W$. "don't care" region from $\frac{2\pi W}{LF_s}$ to $\frac{2\pi}{L} - \frac{2\pi W}{LF_s}$.
- width $= \frac{2\pi}{L}\{1 - \frac{2W}{F_s}\}$.

Session 11 Upsampling & Downsampling

- Narrow "don't care" or transition region requires "long" FIR filter
- See p. 644 in P & M text
- Large number of multiples per output point as well as "large" delay
- Alternatively: Multi-stage interpolation
- Consider 4kHz bandwidth speech sampled at 12 kHz:

- Consider implementing in 3 stages with each stage shown as follows:



Session 11 Upsampling & Downsampling

Let's finish "upsampling" by finishing this example:

•large "don't care"/transition region in each case
•do polyphase implementation at each stage - finish example
• Overall, multistage implementation requires less number of multiples per output point.

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Downsampling:

$$x[n] = x_a\left(\frac{n}{F_s}\right) \longrightarrow \textcircled{\downarrow D} \longrightarrow y[n] = x_a\left(\frac{n}{F_s/D}\right)$$

$$y[n] = x[Dn]$$

e.g.

$$\vdots$$

$$y[-1] = x[-D]$$

$$y[0] = x[0]$$

$$y[1] = x[D]$$

$$y[2] = x[2D]$$

$$\vdots$$

Session 11 Upsampling & Downsampling

- Frequency Domain Analysis:

$Y(\omega) = \sum_{n=-\infty}^{\infty} x[Dn]e^{-j\omega n}$

Let $n = \frac{n'}{d} \implies n' = Dn$

$Y(\omega) = \sum_{n'=mD} x[n']e^{-j\omega \frac{n'}{D}}$

- m is an integer.

$$Y(\omega) = \sum_{n'=-\infty}^{\infty} x[n']\{\sum_{k=0}^{D-1} \frac{1}{D} e^{j\frac{2\pi kn'}{D}}\} e^{-j\frac{\omega}{D}n'} = \begin{cases} 1, & \text{when} \quad n' = mD \\ 0, & \text{otherwise} \end{cases}$$

$Y(\omega) = \frac{1}{D}\sum_{k=0}^{D-1} X(\frac{\omega - k2\pi}{D})$ - Final result.

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