ET 644 Advanced Digital Signal Processing

Homework 1 & 2 Solutions

2.10

The system is nonlinear. This is evident from observation of the pairs

$$x_3(n) \leftrightarrow y_3(n) \text{ and } x_2(n) \leftrightarrow y_2(n).$$

If the system were linear, $y_2(n)$ would be of the form

$$y_2(n) = \{3, 6, 3\}$$

because the system is time-invariant. However, this is not the case.

2.11

since

$$x_1(n) + x_2(n) = \delta(n)$$

and the system is linear, the impulse response of the system is

$$y_1(n) + y_2(n) = \left\{0, \underset{\uparrow}{3}, -1, 2, 1\right\}.$$

If the system were time invariant, the response to $x_3(n)$ would be

$$\left\{\underset{\uparrow}{3}, 2, 1, 3, 1\right\}.$$

But this is not the case.

2.13

A system is BIBO stable if and only if a bounded input produces a bounded output.

$$\begin{aligned} y(n) &= \sum_k h(k)x(n-k) \\ |y(n)| &\leq \sum_k |h(k)||x(n-k)| \\ &\leq M_x \sum_k |h(k)| \end{aligned}$$

where $|x(n-k)| \leq M_x$. Therefore, $|y(n)| < \infty$ for all n, if and only if

$$\sum_k |h(k)| < \infty.$$

2.17

(a)

$$\begin{aligned} x(n) &= \left\{ \underset{\uparrow}{1}, 1, 1, 1 \right\} \\ h(n) &= \left\{ \underset{\uparrow}{6}, 5, 4, 3, 2, 1 \right\} \\ y(n) &= \sum_{k=0}^{n} x(k)h(n-k) \\ y(0) &= x(0)h(0) = 6, \\ y(1) &= x(0)h(1) + x(1)h(0) = 11 \\ y(2) &= x(0)h(2) + x(1)h(1) + x(2)h(0) = 15 \\ y(3) &= x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0) = 18 \\ y(4) &= x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1) + x(4)h(0) = 14 \\ y(5) &= x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(2) + x(4)h(1) + x(5)h(0) = 10 \\ y(6) &= x(1)h(5) + x(2)h(4) + x(3)h(2) = 6 \\ y(7) &= x(2)h(5) + x(3)h(4) = 3 \\ y(8) &= x(3)h(5) = 1 \\ y(n) &= 0, n \geq 9 \\ y(n) &= \left\{ \underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1 \right\} \end{aligned}$$

(b) By following the same procedure as in (a), we obtain

$$y(n) = \left\{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\right\}$$

(c) By following the same procedure as in (a), we obtain

$$y(n) = \left\{1, \underset{\uparrow}{2}, 2, 2, 1\right\}$$

(d) By following the same procedure as in (a), we obtain

$$y(n) = \left\{\underset{\uparrow}{1}, 2, 2, 2, 1\right\}$$

2.18

(a)

$$\begin{aligned}
x(n) &= \left\{\underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\right\} \\
h(n) &= \left\{1, 1, \underset{\uparrow}{1}, 1, 1\right\} \\
y(n) &= x(n) * h(n) \\
&= \left\{\frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\right\}
\end{aligned}$$

(b)

$$\begin{aligned}
x(n) &= \frac{1}{3}n[u(n) - u(n-7)], \\
h(n) &= u(n+2) - u(n-3) \\
y(n) &= x(n) * h(n) \\
&= \frac{1}{3}n[u(n) - u(n-7)] * [u(n+2) - u(n-3)] \\
&= \frac{1}{3}n[u(n) * u(n+2) - u(n) * u(n-3) - u(n-7) * u(n+2) + u(n-7) * u(n-3)] \\
y(n) &= \frac{1}{3}\delta(n+1) + \delta(n) + 2\delta(n-1) + \frac{10}{3}\delta(n-2) + 5\delta(n-3) + \frac{20}{3}\delta(n-4) + 6\delta(n-5) \\
&\quad + 5\delta(n-6) + 5\delta(n-6) + \frac{11}{3}\delta(n-7) + \delta(n-8)
\end{aligned}$$

2.19

$$
\begin{aligned}
y(n) &= \sum_{k=0}^{4} h(k)x(n-k), \\
x(n) &= \left\{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \ldots, \alpha^{5}\right\} \\
h(n) &= \left\{\underset{\uparrow}{1}, 1, 1, 1, 1\right\}
\end{aligned}
$$

$$
\begin{aligned}
y(n) &= \sum_{k=0}^{4} x(n-k), -3 \leq n \leq 9 \\
&= 0, \text{ otherwise.}
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
y(-3) &= \alpha^{-3}, \\
y(-2) &= x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}, \\
y(-1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1}, \\
y(0) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 \\
y(1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha, \\
y(2) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^{2} \\
y(3) &= \alpha^{-1} + 1 + \alpha + \alpha^{2} + \alpha^{3}, \\
y(4) &= \alpha^{4} + \alpha^{3} + \alpha^{2} + \alpha + 1 \\
y(5) &= \alpha + \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}, \\
y(6) &= \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5} \\
y(7) &= \alpha^{3} + \alpha^{4} + \alpha^{5}, \\
y(8) &= \alpha^{4} + \alpha^{5}, \\
y(9) &= \alpha^{5}
\end{aligned}
$$

2.36

$h(n) = a^n u(n)$. The response to $u(n)$ is

$$\begin{aligned}
y_1(n) &= \sum_{k=0}^{\infty} u(k)h(n-k) \\
&= \sum_{k=0}^{n} a^{n-k} \\
&= a^n \sum_{k=0}^{n} a^{-k} \\
&= \frac{1-a^{n+1}}{1-a} u(n) \\
\text{Then, } y(n) &= y_1(n) - y_1(n-10) \\
&= \frac{1}{1-a}\left[(1-a^{n+1})u(n) - (1-a^{n-9})u(n-10)\right]
\end{aligned}$$

2.45

(a)

$$\begin{aligned}
y(n) &= ay(n-1) + bx(n) \\
\Rightarrow h(n) &= ba^n u(n) \\
\sum_{n=0}^{\infty} h(n) &= \frac{b}{1-a} = 1 \\
\Rightarrow b &= 1-a.
\end{aligned}$$

(a)

$$
\begin{aligned}
s(n) &= \sum_{k=0}^{n} h(n-k) \\
&= b\left[\frac{1-a^{n+1}}{1-a}\right]u(n) \\
s(\infty) &= \frac{b}{1-a} = 1 \\
\Rightarrow b &= 1-a.
\end{aligned}
$$

(c) $b = 1 - a$ in both cases.

2.46

(a)

$$
\begin{aligned}
y(n) &= 0.8y(n-1) + 2x(n) + 3x(n-1) \\
y(n) - 0.8y(n-1) &= 2x(n) + 3x(n-1) \\
\text{The characteristic equation is} & \\
\lambda - 0.8 &= 0 \\
\lambda &= 0.8. \\
y_h(n) &= c(0.8)^n
\end{aligned}
$$

Let us first consider the response of the sytem

$$y(n) - 0.8y(n-1) = x(n)$$

to $x(n) = \delta(n)$. Since $y(0) = 1$, it folows that $c = 1$. Then, the impulse response of the original system is

$$
\begin{aligned}
h(n) &= 2(0.8)^n u(n) + 3(0.8)^{n-1}u(n-1) \\
&= 2\delta(n) + 4.6(0.8)^{n-1}u(n-1)
\end{aligned}
$$

(b) The inverse system is characterized by the difference equation

$$x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4y(n-1)$$

Refer to fig 2.6

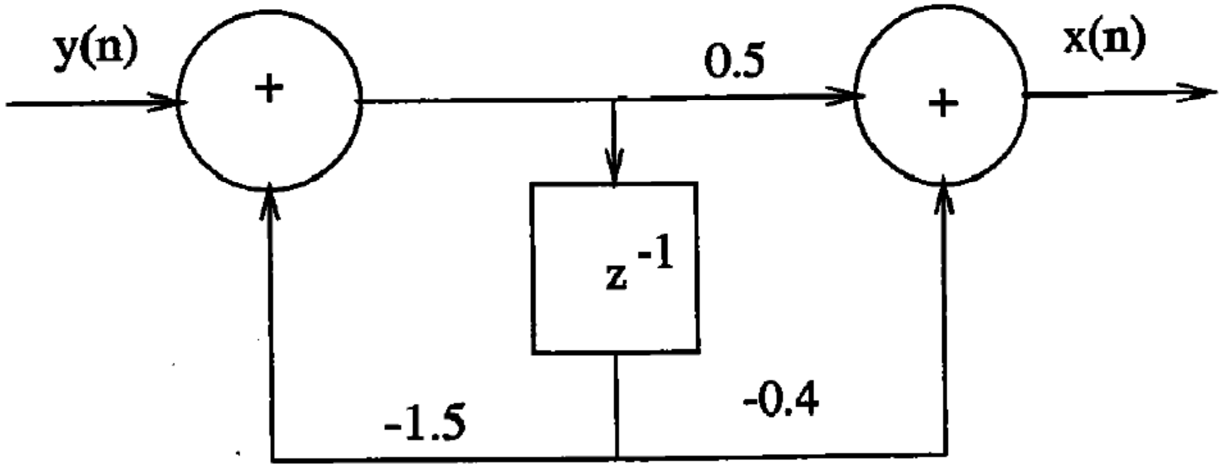

Figure 2.6:

2.61

(a)

$$
\begin{aligned}
\gamma_{xx}(l) &= \sum_{n=-\infty}^{\infty} x(n)x(n-l) \\
&= \sum_{n=-\infty}^{\infty} [s(n)+\gamma_1 s(n-k_1)+\gamma_2 s(n-k_2)] * \\
&\quad [s(n-l)+\gamma_1 s(n-l-k_1)+\gamma_2 s(n-l-k_2)] \\
&= (1+\gamma_1^2+\gamma_2^2)\gamma_{ss}(l)+\gamma_1\left[\gamma_{ss}(l+k_1)+\gamma_{ss}(l-k_1)\right] \\
&\quad +\gamma_2\left[\gamma_{ss}(l+k_2)+\gamma_{ss}(l-k_2)\right] \\
&\quad +\gamma_1\gamma_2\left[\gamma_{ss}(l+k_1-k_2)+\gamma_{ss}(l+k_2-k_1)\right]
\end{aligned}
$$

(b) $\gamma_{xx}(l)$ has peaks at $l=0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$. Suppose that $k_1 < k_2$. Then, we can determine γ_1 and k_1. The problem is to determine γ_2 and k_2 from the other peaks.
(c) If $\gamma_2 = 0$, the peaks occur at $l=0$ and $l=\pm k_1$. Then, it is easy to obtain γ_1 and k_1.

2.62

(a)
(b) variance = 0.01. Refer to fig 2.7.
(b) Delay D = 20. Refer to fig 2.7.

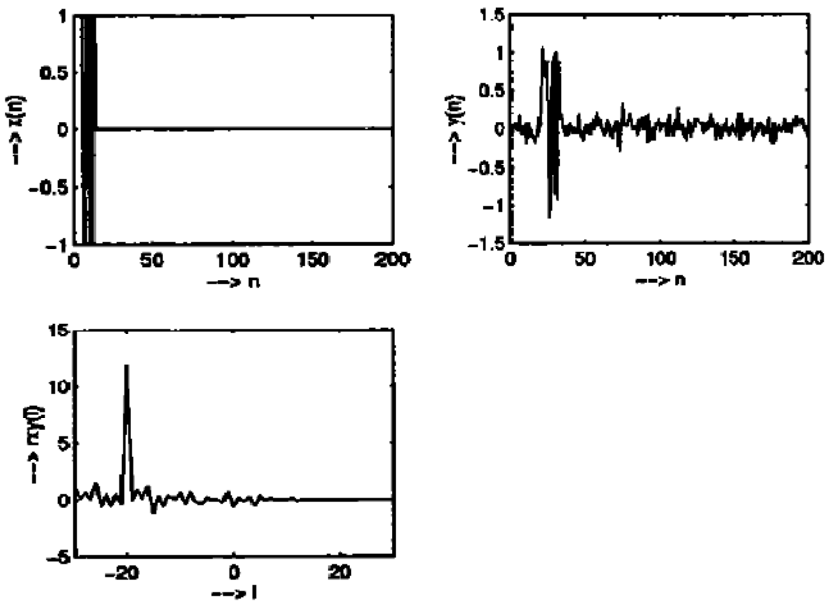

Figure 2.7: variance = 0.01

(c) variance = 0.1. Delay D = 20. Refer to fig 2.8.
(d) Variance = 1. delay D = 20. Refer to fig 2.9.

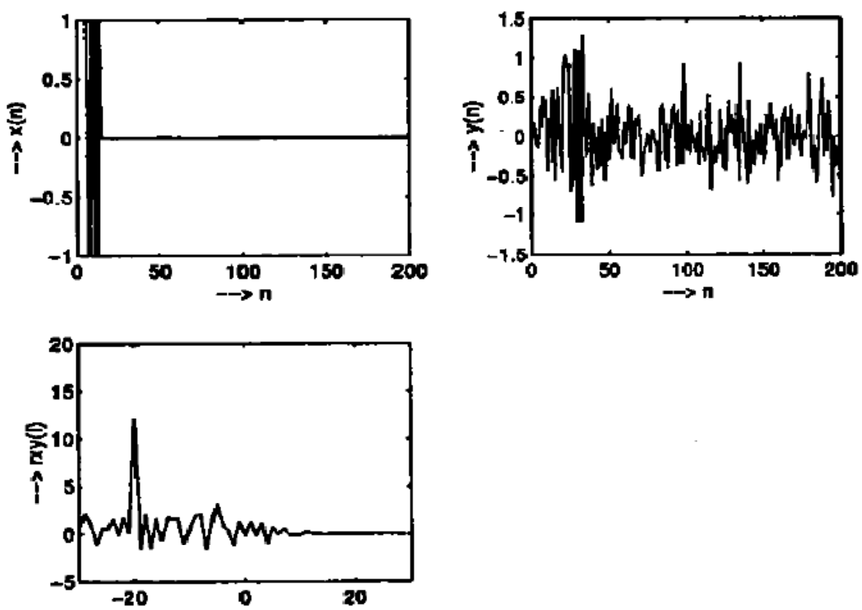

Figure 2.8: variance = 0.1

(e) $x(n) = \{-1, -1, -1, +1, +1, +1, +1, -1, +1, -1, +1, +1, -1, -1, +1\}$. Refer to fig 2.10.
(f) Refer to fig 2.11.

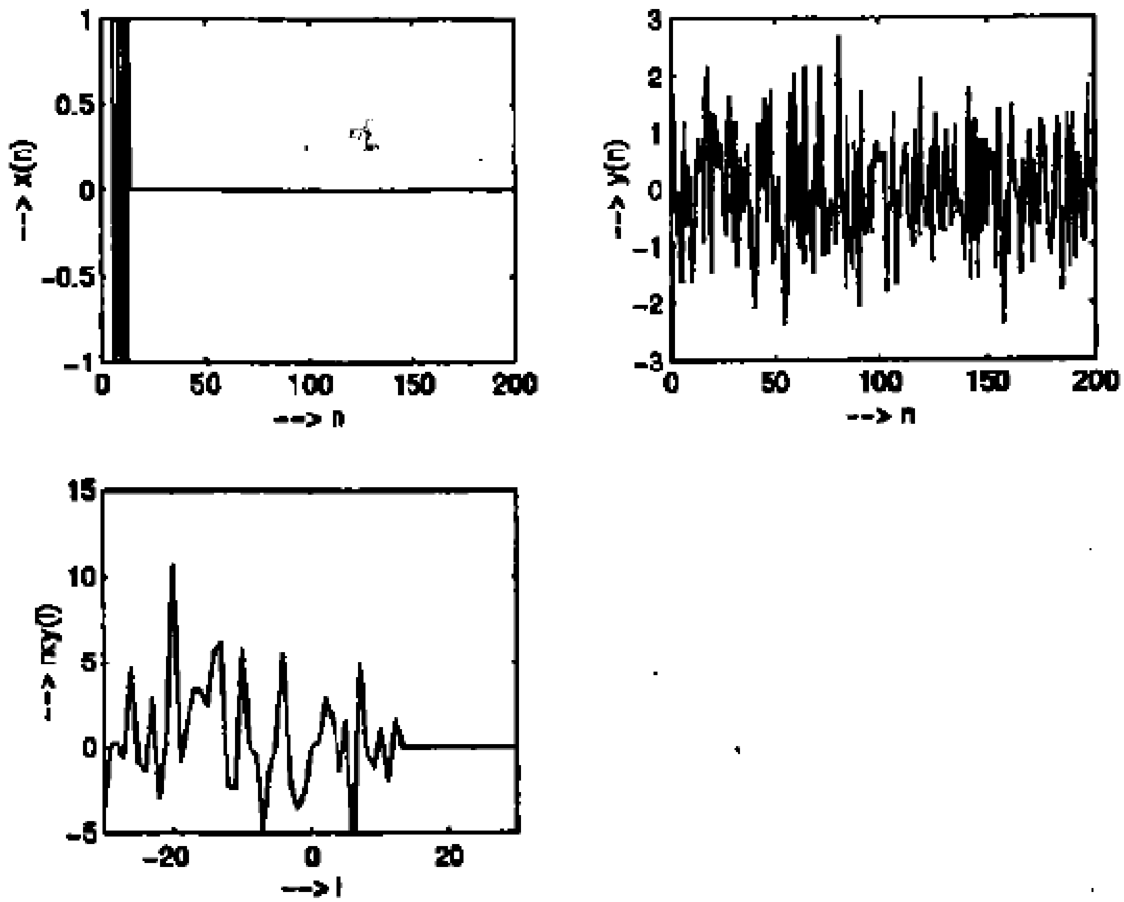

Figure 2.9: variance = 1

2.63

(a) Refer to fig 2.12.
(b) Refer to fig 2.13.
(c) Refer to fig 2.14.
(d) The step responses in fig 2.13 and fig 2.14 are similar except for the steady state value after n=20.

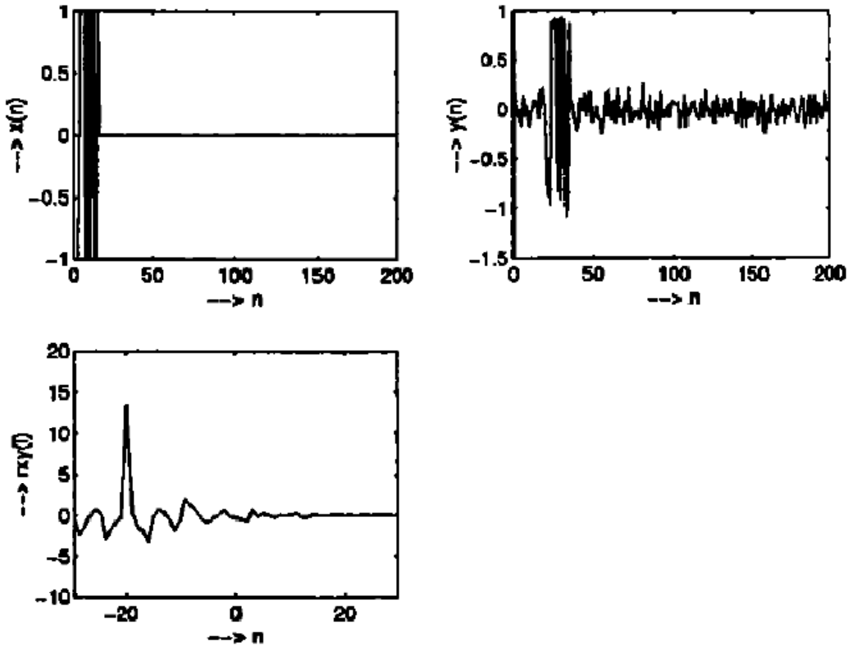

Figure 2.10:

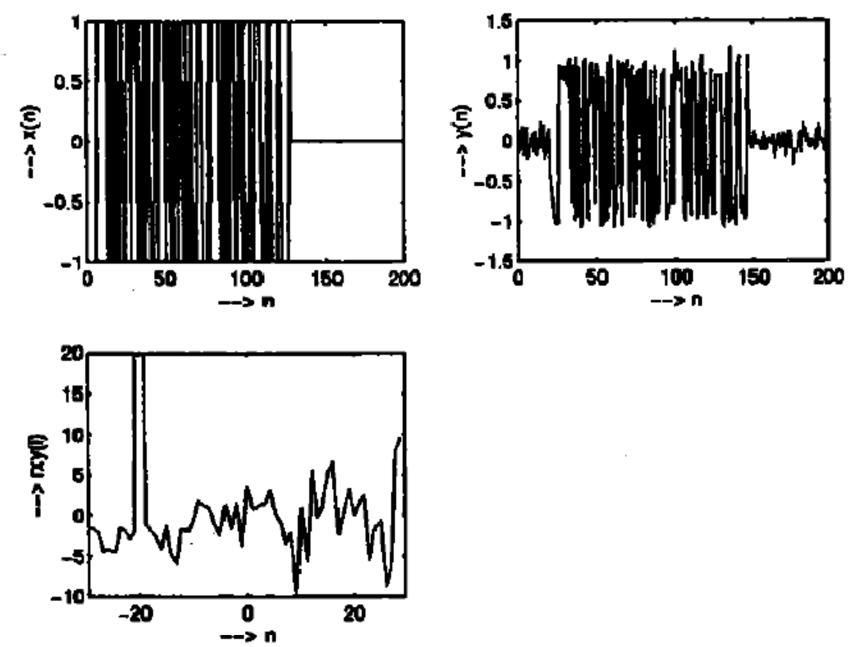

Figure 2.11:

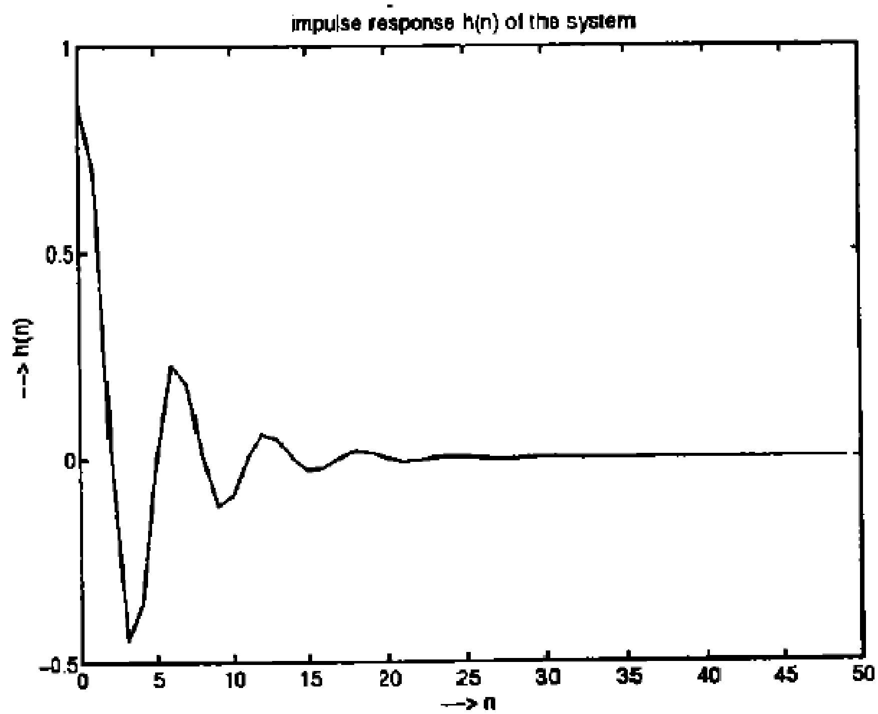

Figure 2.12:

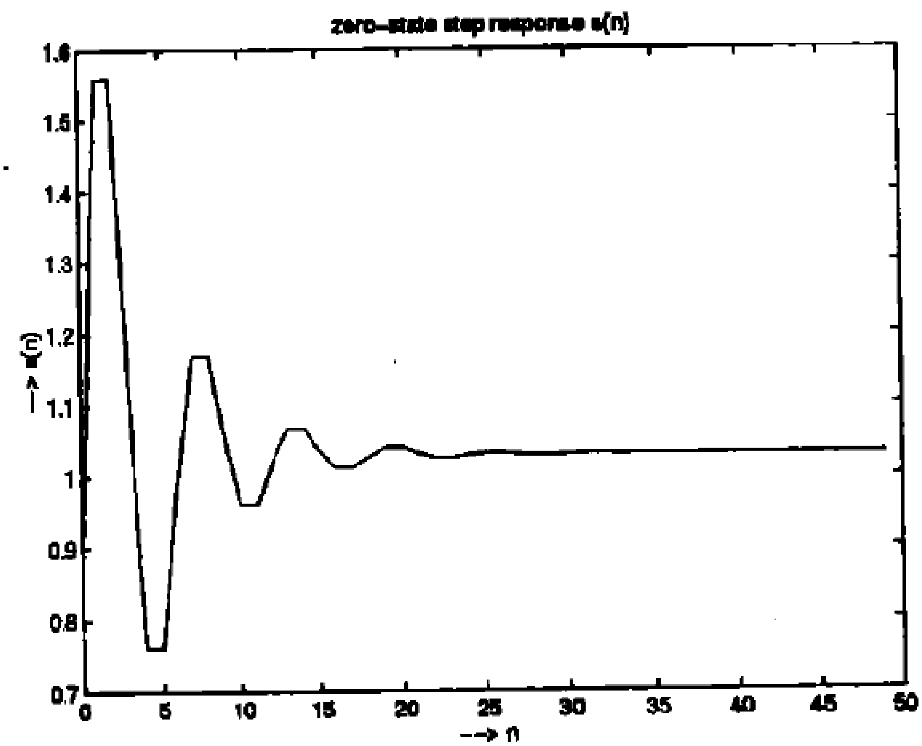

Figure 2.13:

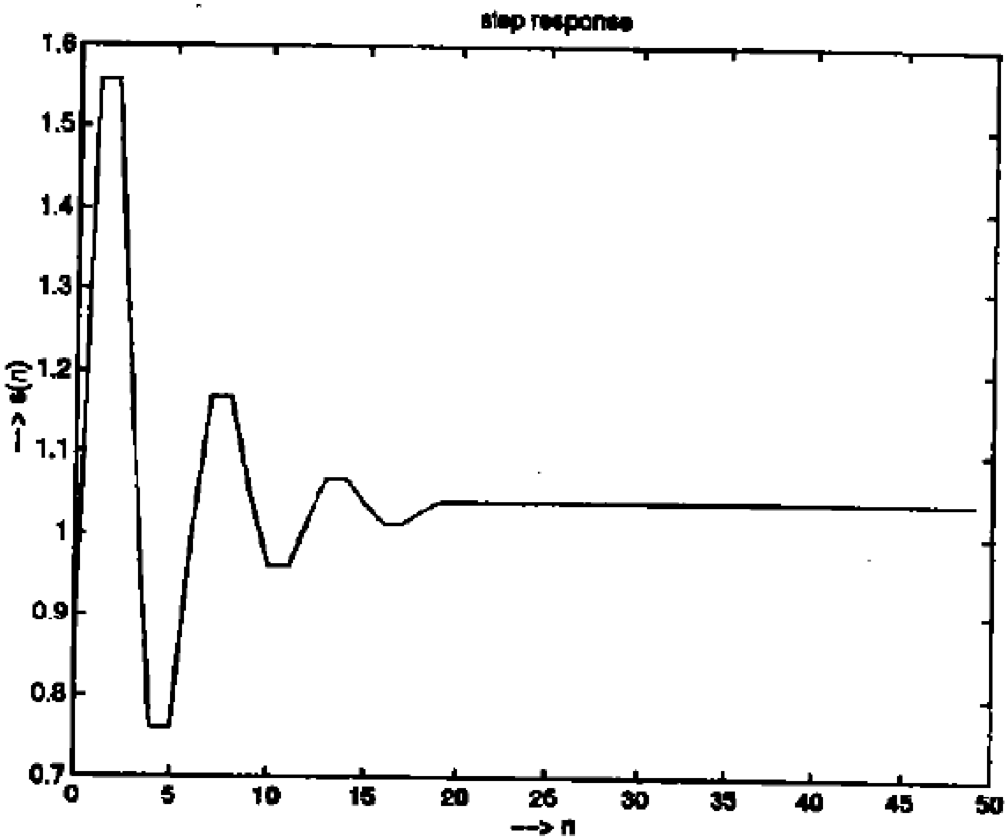
step response
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
---> s(n)
0
5
10
15
20
25
30
35
40
45
50
---> n

Figure 2.14: