TOPIC 47 THE MEAN AND STANDARD DEVIATION

Often, a distribution of scores is described with only two statistics: the **mean** to describe its *average* and the **standard deviation** (whose symbol is *S* or *SD* for a population, and *s* or *sd* for a sample) to describe its *variability*. The term "variability" refers to the amount by which participants *vary* or differ from each other. Let us see what this means by considering the scores of three groups, all of which have the same mean but different standard deviations.

Group A: 0, 5, 10, 15, 20, 25, 30
$M = 15.00, S = 10.00$

Group B: 14, 14, 14, 15, 16, 16, 16
$M = 15.00, S = 0.93$

Group C: 15, 15, 15, 15, 15, 15, 15
$M = 15.00, S = 0.00$

Although Groups A, B, and C are the same on the average, as indicated by the mean, they are very different in terms of variability. Notice that the differences among the scores of Group A (a score of 0 vs. a score of 5 vs. a score of 10 vs. a score of 15, and so on) are much greater than the differences among the scores of Group B (a score of 14 vs. a score of 14 vs. a score of 14 vs. a score of 15, and so on). At the extreme, when all the scores are the same, as in Group C, there is no variability. As a result, the standard deviation equals zero. Thus, as a rule, the smaller the variability, the smaller the standard deviation.[1]

The standard deviation has a special relationship to the normal curve (see Topic 45). *If a distribution is normal, 68% of the participants in the distribution lie within one standard-deviation unit of the mean.*[2] For example, if you read in a report that $M = 70$ and $S = 10$ for a normal distribution, you would know that 68% of the participants have scores between 60 and 80 (i.e., 70 – 10 = 60 and 70 + 10 = 80). This is illustrated in Figure 1.

In Figure 2, the mean is also 70, but the standard deviation is only 5. The smaller standard deviation in Figure 2 is reflected by the fact that the curve is narrower than in Figure 1. Yet in both distributions, 68% of the cases lies within one standard deviation unit of the mean because they are both normal.

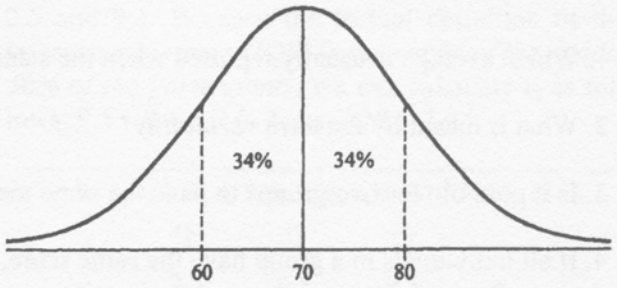

Figure 1. Normal curve with a mean of 70 and a standard deviation of 10.

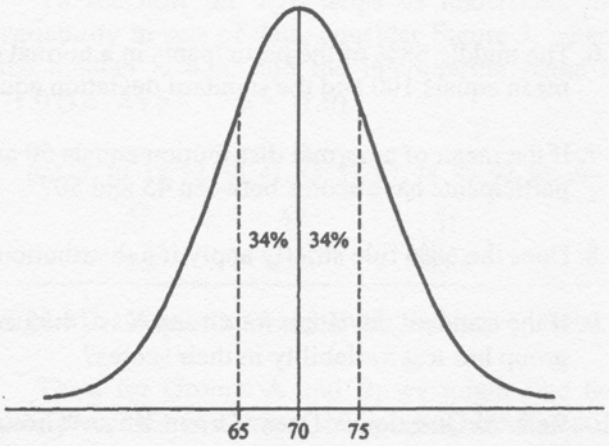

Figure 2. Normal curve with a mean of 70 and a standard deviation of 5.

Note that the middle area of Figure 2 is more narrow *but taller* than the middle area of Figure 1. This additional height in Figure 2 makes it possible for it to have the same percentage (68%) of cases as the middle area of Figure 1 has.

At first, this may seem like magic. Regardless of the value of the standard deviation, 68% of the cases lies within one standard-deviation unit in a normal curve. Actually, it is not magic but a property of the normal curve. When you are calculating the standard deviation, you are actually calculating the number of points that one must go out from the mean to capture 68% of the cases. This 68% rule does *not* strictly apply if the distribution is *not* normal. The less normal it is, the less accurate the rule is. Put another way, if the middle area of a distribution does not contain approximately 68% of the cases, the distribution is not normal.

[1] For those of you who are mathematically inclined, the computation of the standard deviation is illustrated in Appendix F. Considering the method of computation may give you a better understanding of its meaning.

[2] Note that *within* means on *both sides of the mean* (i.e., the mean plus/minus the standard deviation).

Note that the normal curve (see Topic 45) is not an invention of statisticians. Instead, it is a curve that has been observed with great frequency in nature. Statisticians derived the standard deviation in order to have a standardized method for describing the variability of normal distributions.

EXERCISE ON TOPIC 47

1. Which average is usually reported when the standard deviation is reported?

2. What is meant by the term *variability*?

3. Is it possible for two groups to have the same mean but different standard deviations?

4. If all individuals in a group have the same score, what is the value of the standard deviation for the scores?

5. What percentage of the participants lies within one standard-deviation unit of the mean (i.e., on both sides of the mean) in a normal distribution?

6. The middle 68% of the participants in a normal distribution have scores between what two values if the mean equals 100 and the standard deviation equals 15?

7. If the mean of a normal distribution equals 50 and the standard deviation equals 5, what percentage of the participants have scores between 45 and 50?

8. Does the 68% rule strictly apply if a distribution is *not* normal?

9. If the standard deviation for Group X is 14.55 and the standard deviation for Group Y is 20.99, which group has less variability in their scores?

10. Refer to Question 9. Does "Group X" *or* "Group Y" have a narrower curve?

Question for Discussion

11. Locate a journal article in which the researcher reports a mean and standard deviation. Does the researcher indicate whether the underlying distribution being described is normal in shape? Do you think the 68% rule strictly applies? Explain.

For Students Who Are Planning Research

12. Will you be reporting means and standard deviations? Explain.