Let $\alpha$, $\beta$ and $\gamma$ denote the 1st, 2nd and 6th integers in your social security number.

After years of constant civil war, the small (imaginary) Eastern European country of Tolkeinia is experiencing a period of relative calm. The country’s infrastructure is non-existent. Assisted by the United Nations, Tolkeinia is in the process of re-establishing a telecommunications network. Due to the ruggedness of the terrain, and cost considerations, the government has decided to develop a wireless communications network. This requires the positioning of transmitting and repeater towers through the country. Amazingly, the country is semi-elliptical in shape, and satellite data has been interpolated to obtain elevation profiles for the land.

Your company has been contracted to analyze the data and suggest appropriate locations for two towers. Due to economic development considerations these towers need to be distributed throughout the country.

Assume that the vertical elevation (in feet above sea level) is given by:

$$h(x, y) = 10.\alpha \times \left(x^3 + y^5 + \frac{x}{5,\beta}\right) \times \exp(-x^2 - y^2) + \frac{\exp(-(x - 1)^2 - y^2)}{3.\gamma}$$

for $0 \leq x^2 + 2y^2 \leq 9, \; y \geq 0$.

Questions:

This project should be treated as a report to your supervisors. Be very thorough in your work, giving sufficient evidence supporting your work (including graphs of surfaces and functions). Make sure that your explanations are clear and concise, and provide enough information for your supervisor to follow your work. [Your write up will count as 20 % of your final grade.]

1. Present several (at least two) different views of the surface [Hint: Due to the elliptical domain, it is probably best to convert to cylindrical coordinates when plotting the surface.]

2. Include a contour plot of Tolkeinia.

3. Locate the interior maxima and minima.

4. Check for maxima and minima on the boundary.

5. Discuss and justify your recommendations for the locations of the towers in Tolkeinia.