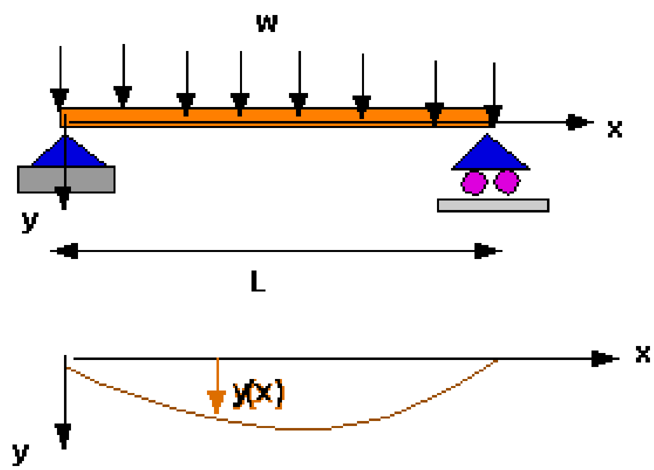
Introduction to Beam Theory

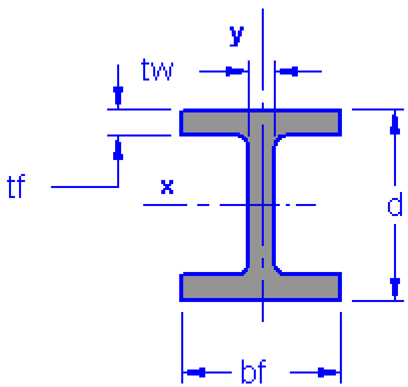
Area Moments of Inertia, Deflection, and Volumes of Beams



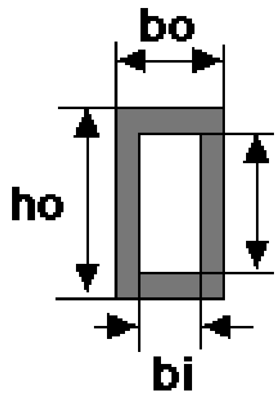
What is a Beam?

- Horizontal structural member used to support horizontal loads such as floors, roofs, and decks.
- Types of beam loads
  - Uniform
  - Varied by length
  - Single point
  - Combination

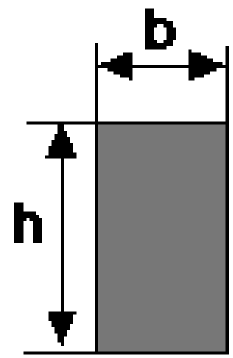
Common Beam Shapes



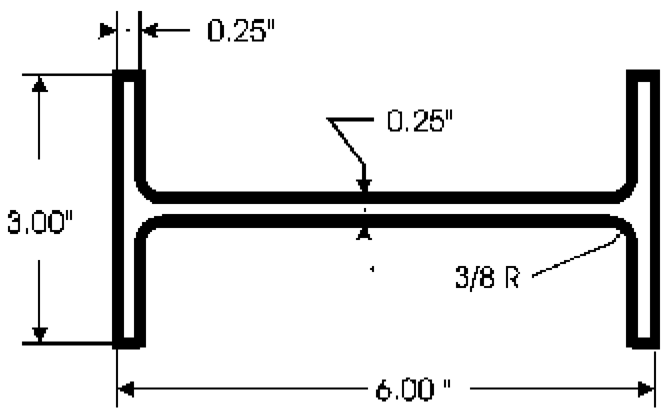
I Beam



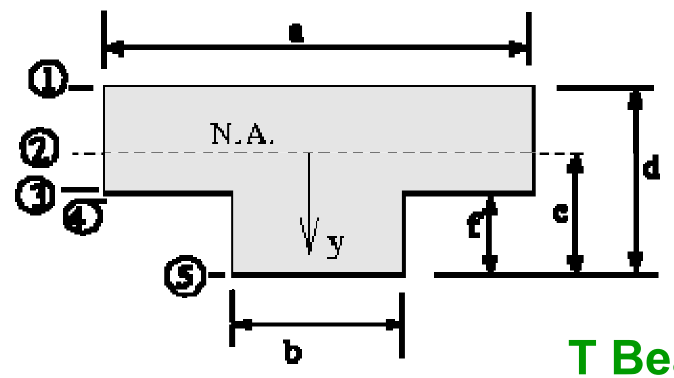
Hollow Box



Solid Box



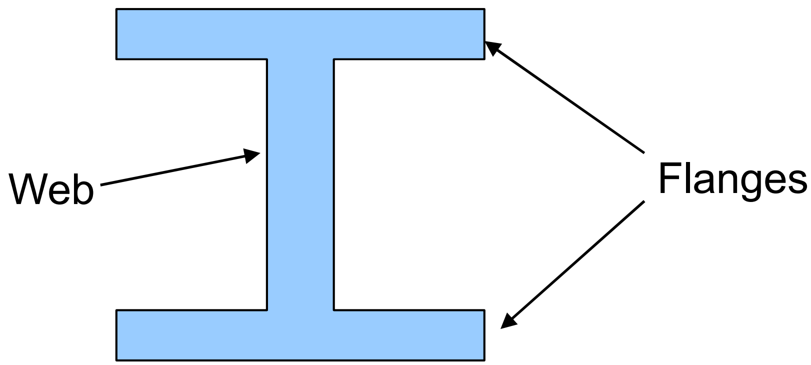
H Beam

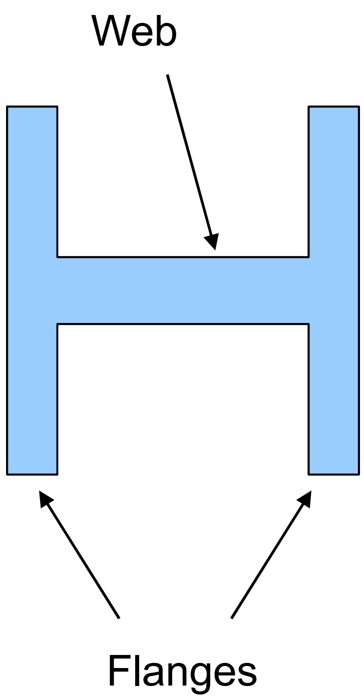


T Beam

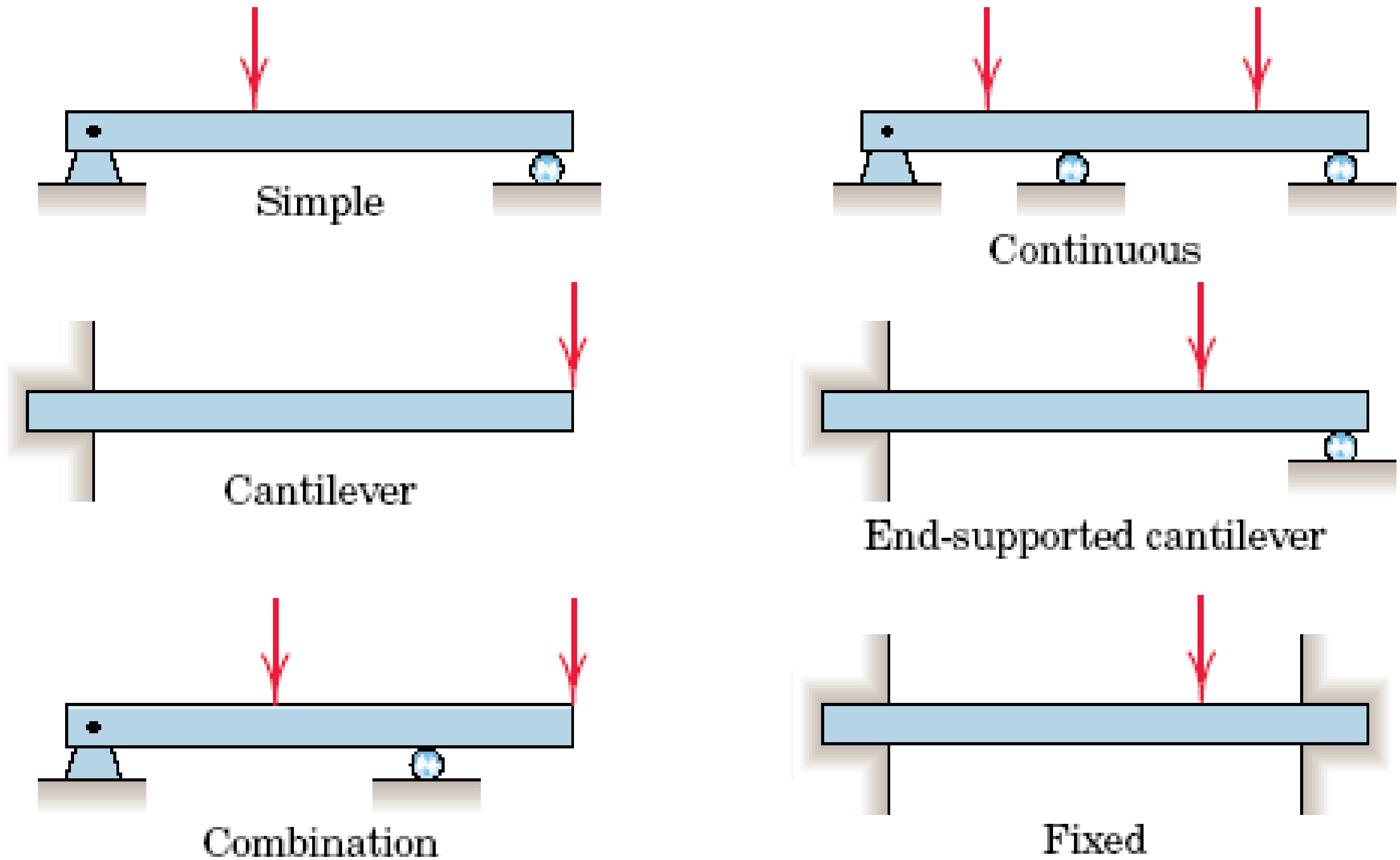
Beam Terminology

- The parallel portions on an I-beam or H-beam are referred to as the ***flanges***. The portion that connects the flanges is referred to as the ***web***.



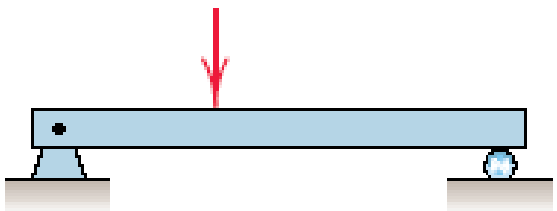


Support Configurations

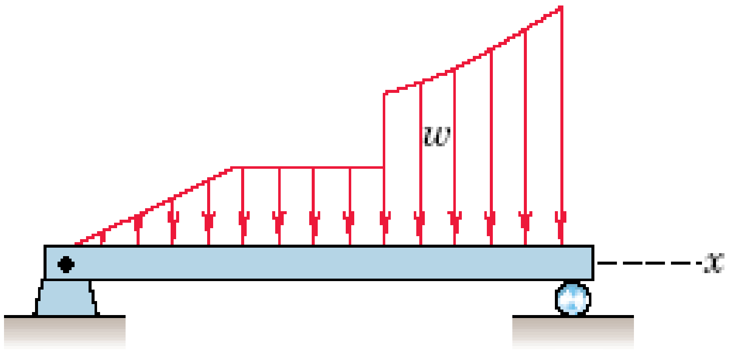


Source: ***Statics*** (Fifth Edition), Meriam and Kraige, Wiley

Load and Force Configurations



Concentrated Load

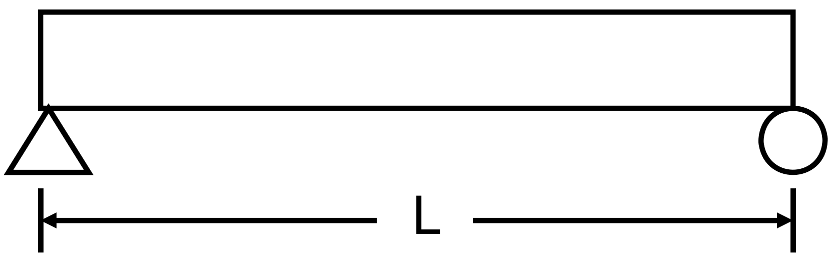


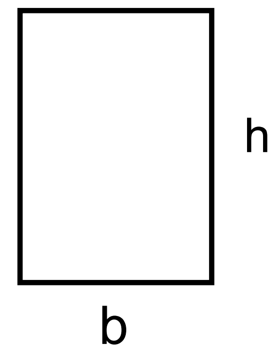
Distributed Load

Source: ***Statics*** (Fifth Edition), Meriam and Kraige, Wiley

Beam Geometry

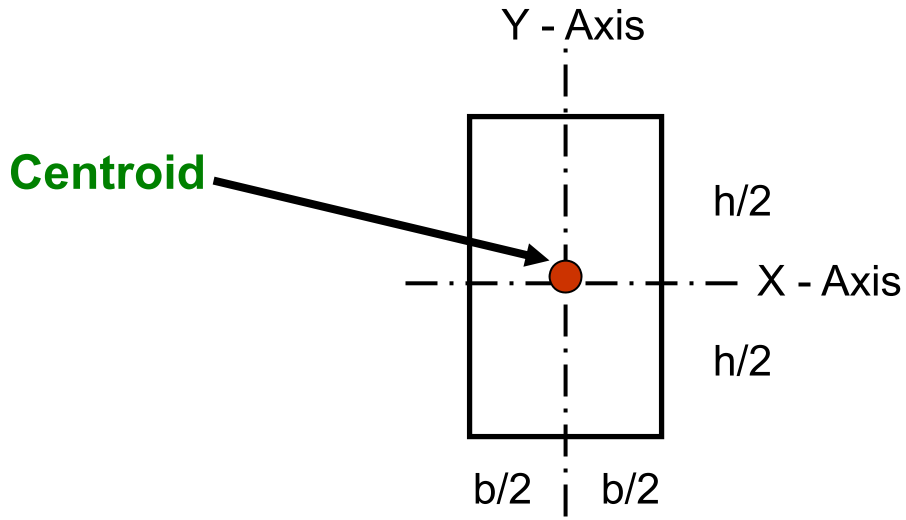
- Consider a simply supported beam of length, L.
- The cross section is rectangular, with width, b, and height, h.





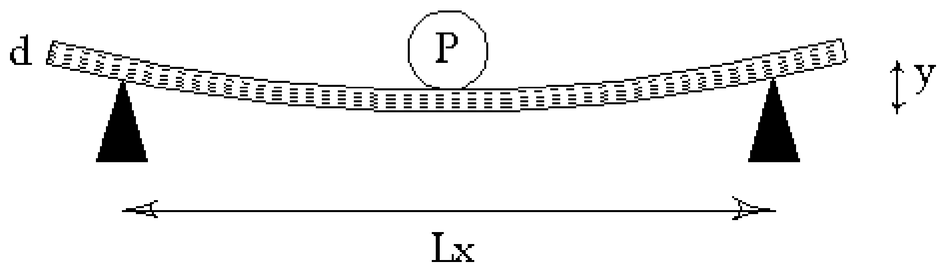
Beam Centroid

- An area has a centroid, which is similar to a center of gravity of a solid body.
- The centroid ● of a symmetric cross section can be easily found by inspection. X and Y axes intersect at the centroid of a symmetric cross section, as shown on the rectangular cross section.



Area Moment of Inertia (I)

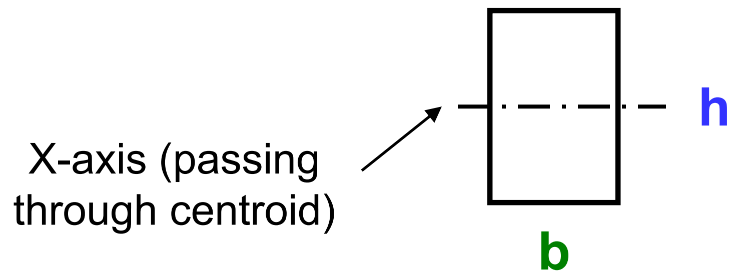
- **Inertia** is a measure of a body's ability to resist movement, bending, or rotation
- **Moment of inertia (I)** is a measure of a beam's
  - Stiffness with respect to its cross section
  - Ability to resist bending
- As **I** increases, bending decreases
- As **I** decreases, bending increases
- Units of **I** are $(\text{length})^4$, e.g. in^4, ft^4, or cm^4



I for Common Cross-Sections

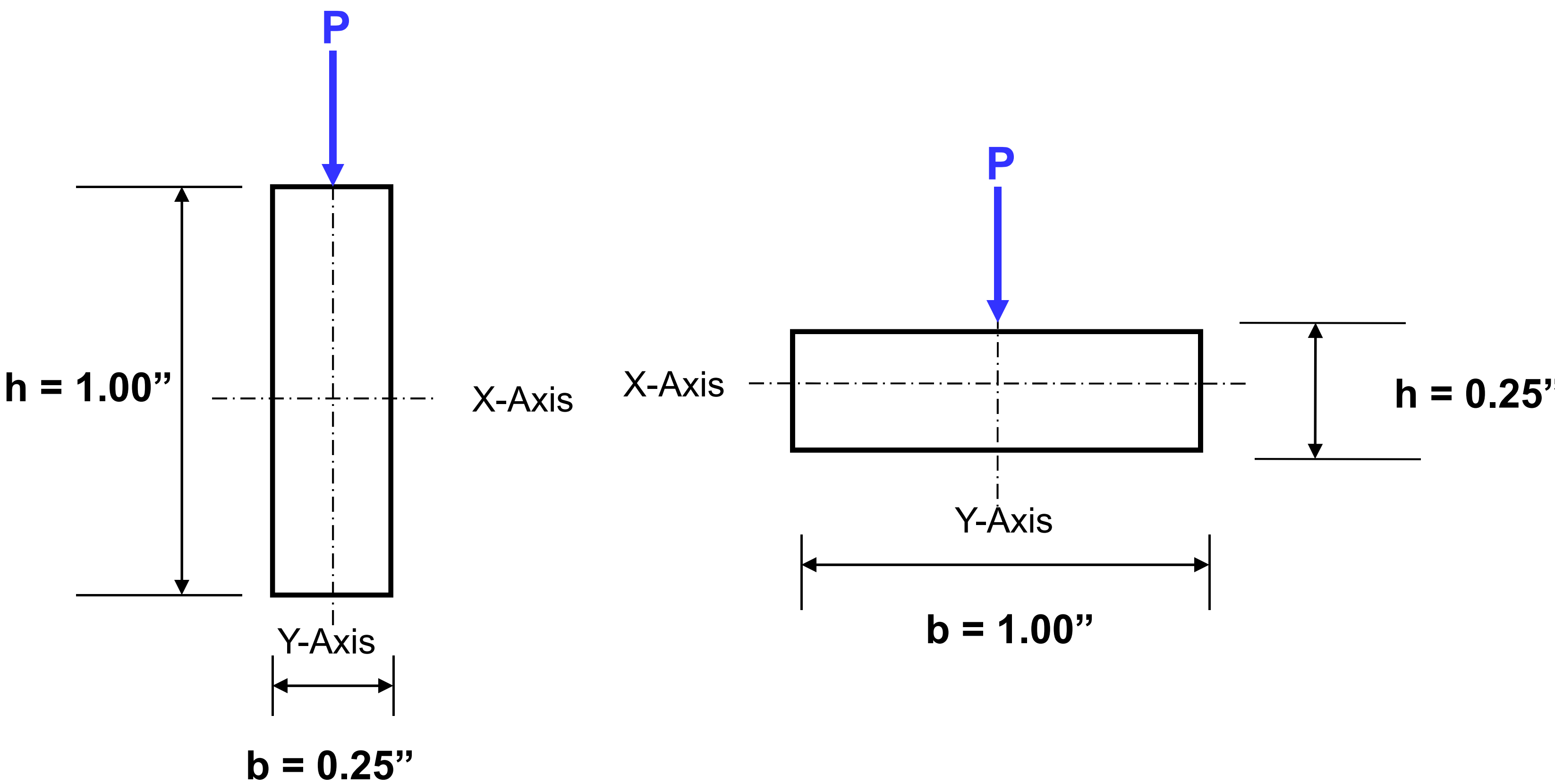
- **I** can be derived for any common area using calculus. However, moment of inertia equations for common cross sections (e.g., rectangular, circular, triangular) are readily available in math and engineering textbooks.
- For a **solid rectangular** cross section,

$$I_x = \frac{bh^3}{12}$$



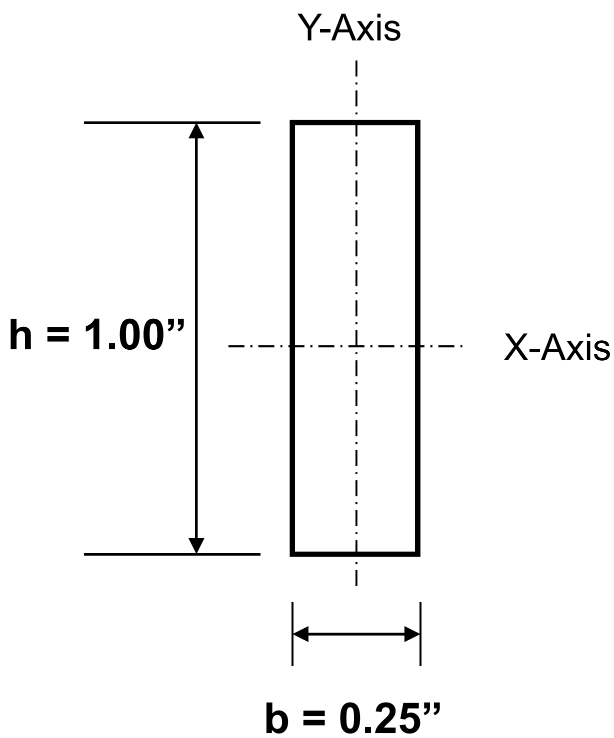
- **b** is the dimension **parallel** to the bending axis
- **h** is the dimension **perpendicular** to the bending axis

Which Beam Will Bend (or Deflect) the Most About the X-Axis?



Solid Rectangular Beam #1

- Calculate the moment of inertia about the X-axis



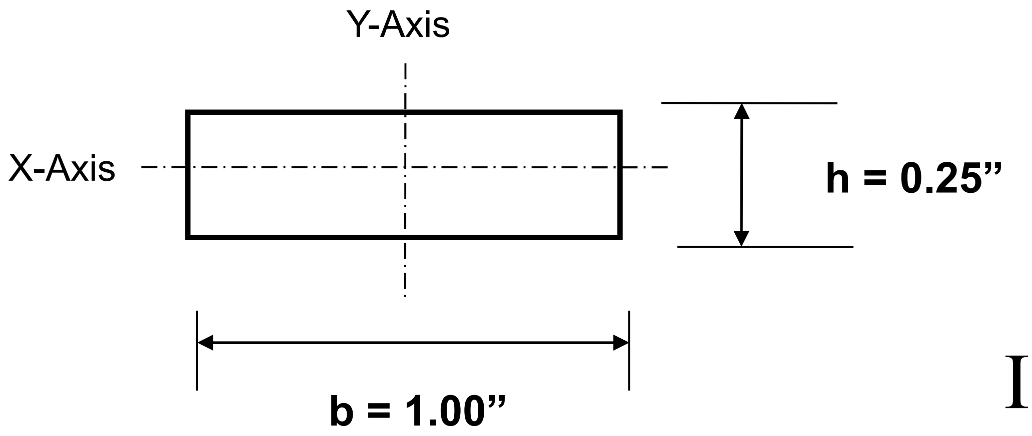
$$I_x = \frac{bh^3}{12}$$

$$I_x = \frac{(0.25\ \text{in})(1.00\ \text{in})^3}{12}$$

$$I_x = 0.02083\ \text{in}^4$$

Solid Rectangular Beam #2

- Calculate the moment of inertia about the X-axis

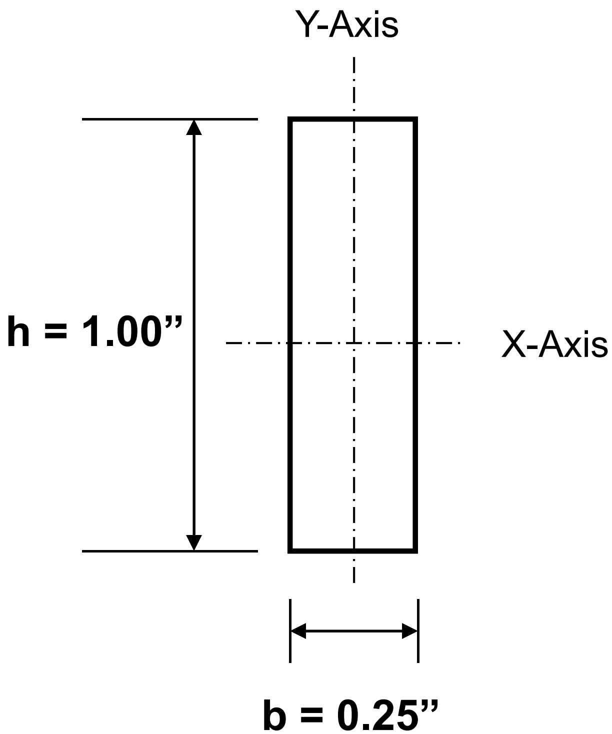


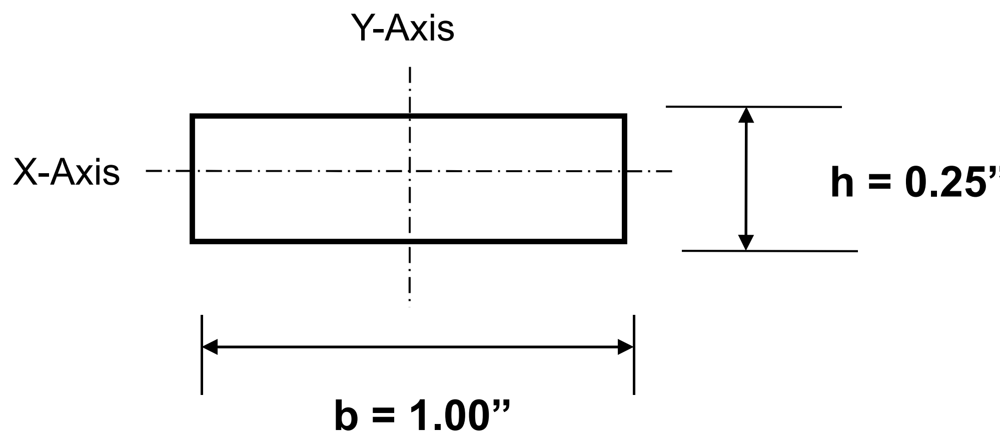
$$I_X = \frac{bh^3}{12}$$

$$I_X = \frac{(1.00\text{ in})(0.25\text{ in})^3}{12}$$

$$I_X = 0.00130\text{ in}^4$$

Compare Values of I_x





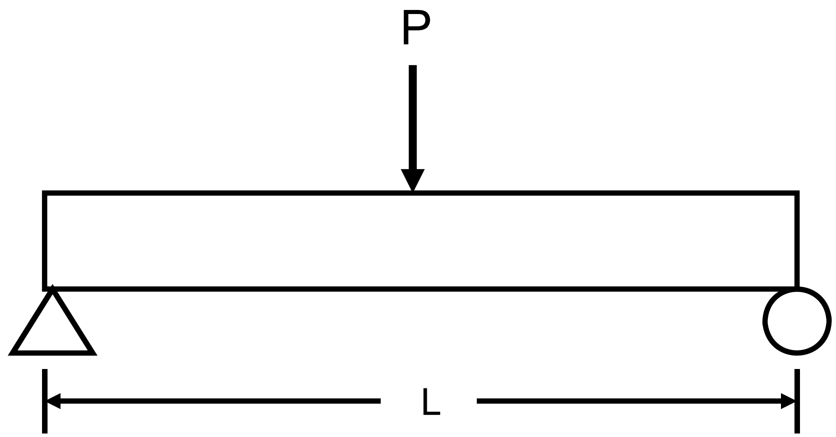
$$I_x = 0.02083 \text{ in}^4$$

$$I_X = 0.00130 \text{ in}^4$$

Which beam will bend or deflect the most? Why?

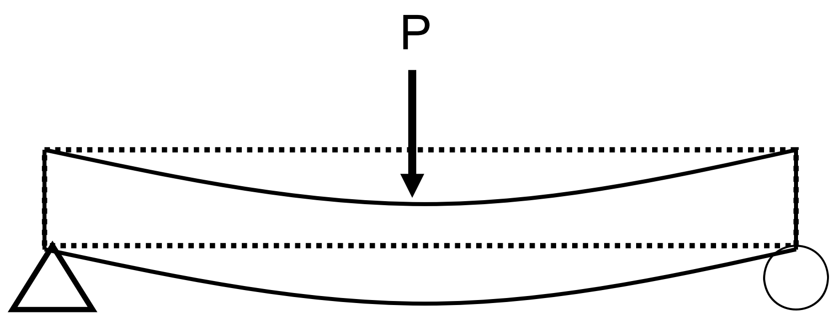
Concentrated ("Point") Load

- Suppose a concentrated load, P (lb_f), is applied to the center of the simply supported beam



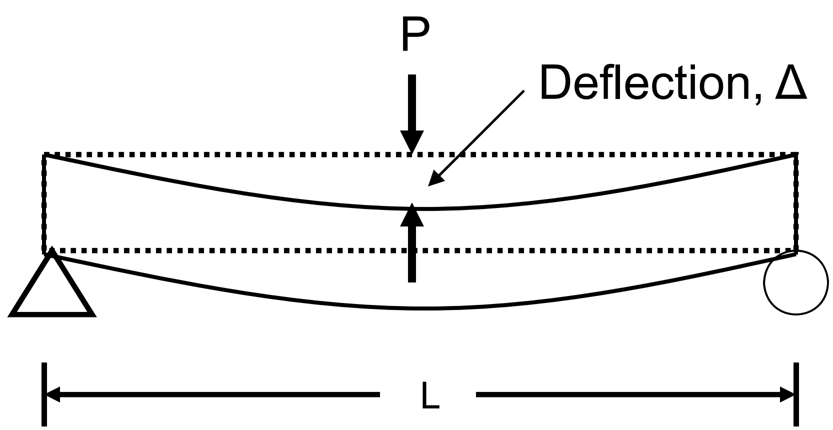
Deflection

- The beam will bend or deflect downward as a result of the load P (lb_f).



Deflection (Δ)

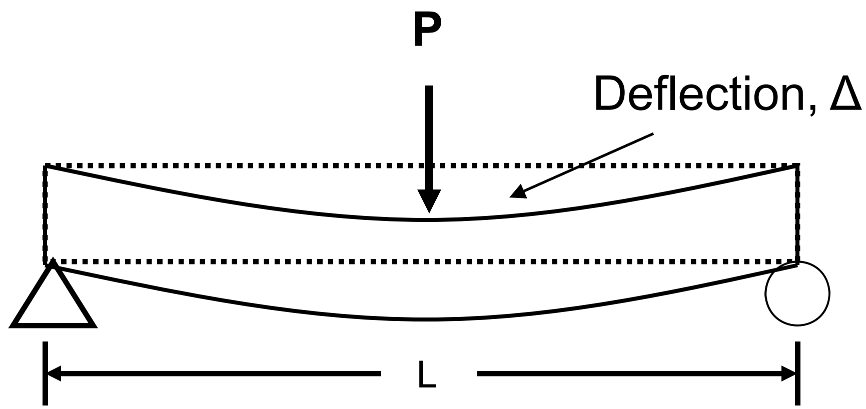
- Δ is a measure of the vertical displacement of the beam as a result of the load P (lb_f).



Deflection (Δ)

- Δ of a simply supported, center loaded beam can be calculated from the following formula:

$$\Delta = \frac{PL^3}{48EI}$$



P = concentrated load (**lb_f**)

L = span length of beam (in)

E = modulus of elasticity (psi or lb_f/in^2)

I = moment of inertia of axis **perpendicular to load P** (in^4)

Deflection (Δ)

$$\Delta = \frac{PL^3}{48EI}$$

I, the Moment of Inertia, is a significant variable in the determination of beam deflection

But….What is E?

Modulus of Elasticity (E)

- Material **property** that indicates stiffness and rigidity
- Values of **E** for many materials are readily available in tables in textbooks.
- Some common values are

Material	E (psi)
Steel	30×10^6
Aluminum	10×10^6
Wood	$\sim 2 \times 10^6$

Consider…

If the cross-sectional area of a solid wood beam is enlarged, how does the **Modulus of Elasticity, E,** change?

$$\Delta = \frac{PL^3}{48EI} \qquad I_x = \frac{bh^3}{12}$$

Material	E (psi)
Steel	30×10^6
Aluminum	10×10^6
Wood	$\sim 2 \times 10^6$

Consider…

Assuming the same rectangular cross-sectional area, which will have the larger **Moment of Inertia, I,** steel or wood?

$$\Delta = \frac{PL^3}{48EI} \qquad I_x = \frac{bh^3}{12}$$

Material	E (psi)
Steel	30×10^6
Aluminum	10×10^6
Wood	$\sim 2 \times 10^6$

Consider…

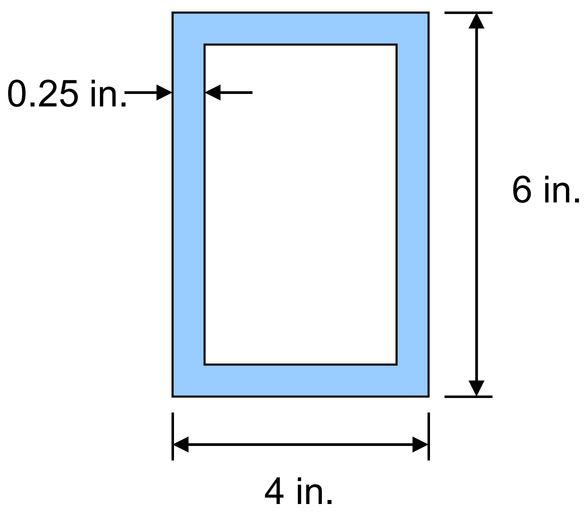
Assuming beams with the same cross-sectional area and length, which will have the larger **deflection, Δ,** steel or wood?

$$\Delta = \frac{PL^3}{48EI} \qquad I_x = \frac{bh^3}{12}$$

Material	E (psi)
Steel	30×10^6
Aluminum	10×10^6
Wood	$\sim 2 \times 10^6$

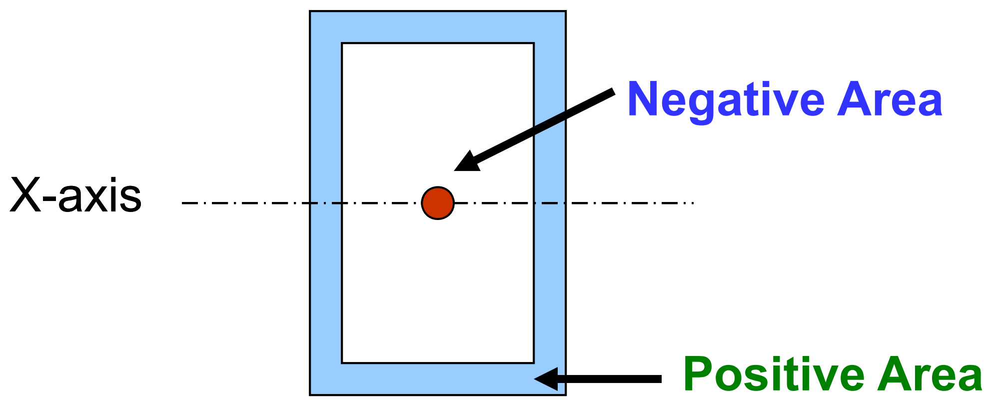
More Complex Designs

- The calculations for Moment of Inertia are very simple for a solid, symmetric cross section.
- Calculating the moment of inertia for more complex cross-sectional areas takes a little more effort.
- Consider a hollow box beam as shown below:



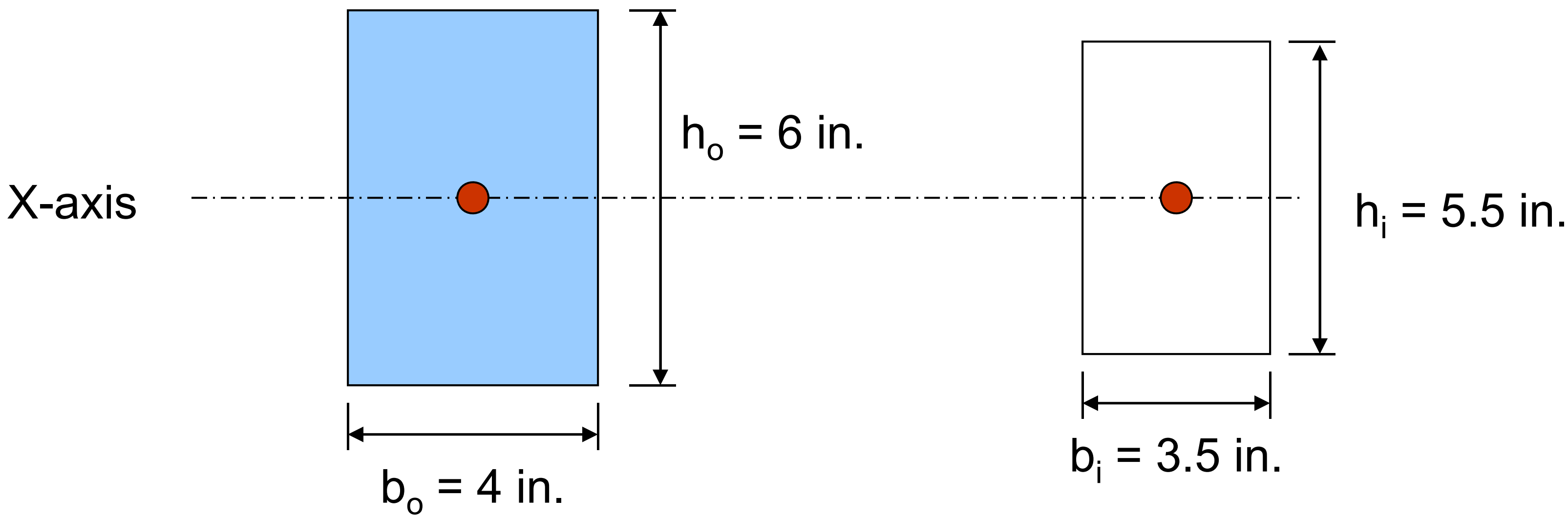
Hollow Box Beams

- The same equation for moment of inertia, $I = bh^3/12$, can be used but is used in a different way.
- Treat the outer dimensions as a ***positive area*** and the inner dimensions as a ***negative area***, as the centroids of both are about the same X-axis.

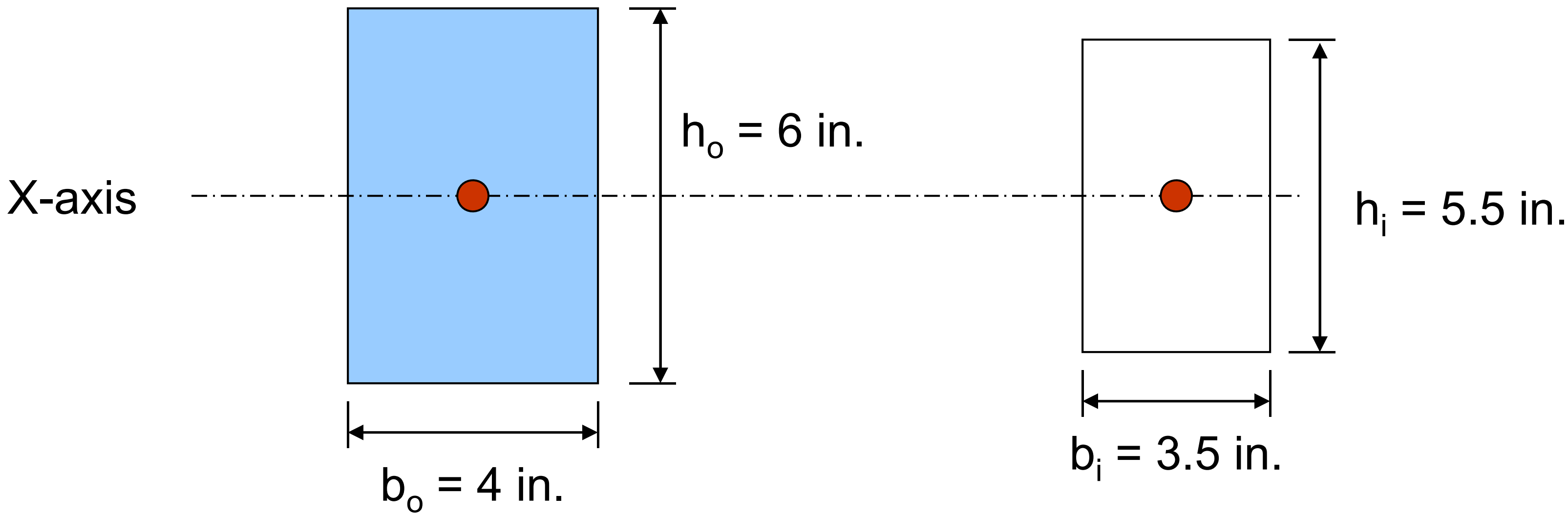


Hollow Box Beams

- Calculate the moment of inertia about the X-axis for the positive area and the negative area using $I = bh^3/12$.
- The outer dimensions will be denoted with subscript "o" and the inner dimensions will be denoted with subscript "i".



Hollow Box Beams



$$I_{pos} = \frac{b_o\, h_o^{\,3}}{12}$$

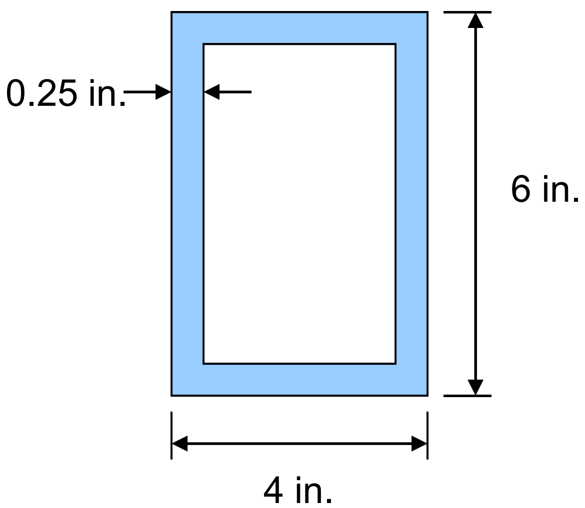
$$I_{neg} = \frac{b_i\, h_i^{\,3}}{12}$$

$$I_{pos} = \frac{(4\text{ in})(6\text{ in})^3}{12}$$

$$I_{neg} = \frac{(3.5\text{ in})(5.5\text{ in})^3}{12}$$

Hollow Box Beams

- Simply subtract I_{neg} from I_{pos} to calculate the moment of inertia of the box beam, I_{box}



$$I_{box} = I_{pos} - I_{neg}$$

$$I_{box} = \frac{b_o h_o^3}{12} - \frac{b_i h_i^3}{12}$$

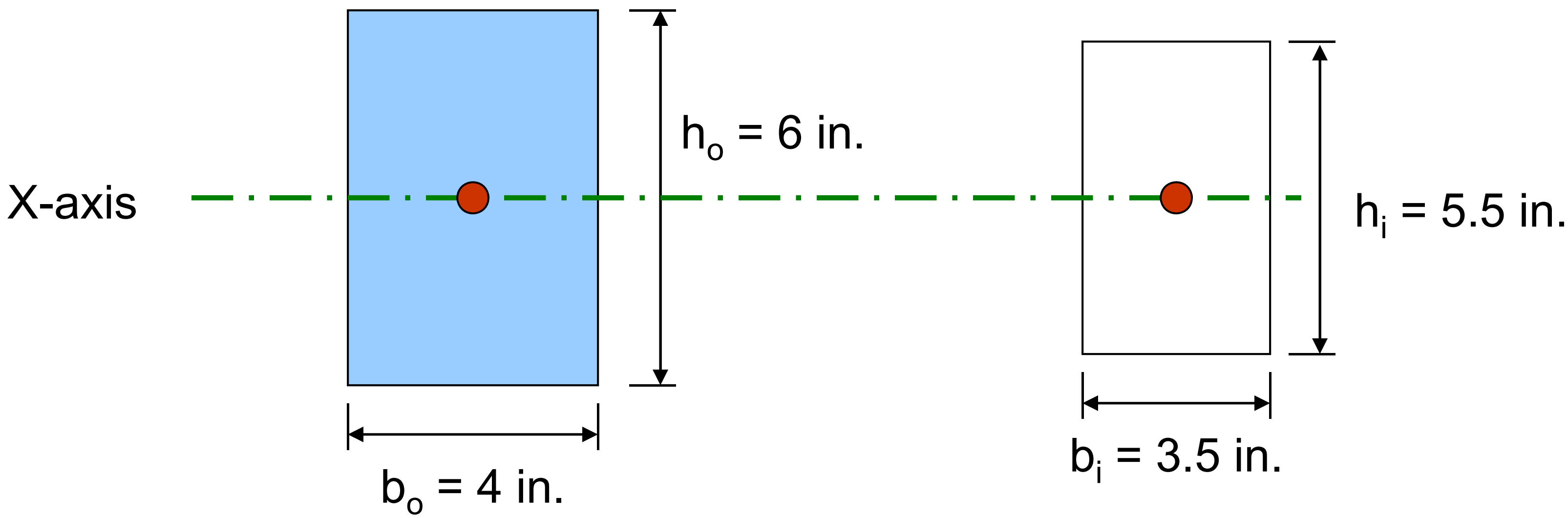
$$I_{box} = \frac{(4\text{ in})(6\text{ in})^3}{12} - \frac{(3.5\text{ in})(5.5\text{ in})^3}{12}$$

$$I_{box} = \frac{(4\text{ in})(216\text{ in}^3)}{12} - \frac{(3.5\text{ in})(166.4\text{ in}^3)}{12}$$

$$I_{box} = 23.5\ \text{in}^4$$

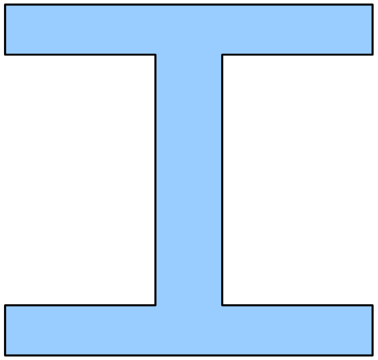
Important

- In order to use the "positive-negative area" approach, the centroids ● of both the positive and negative areas must be on the same axis!



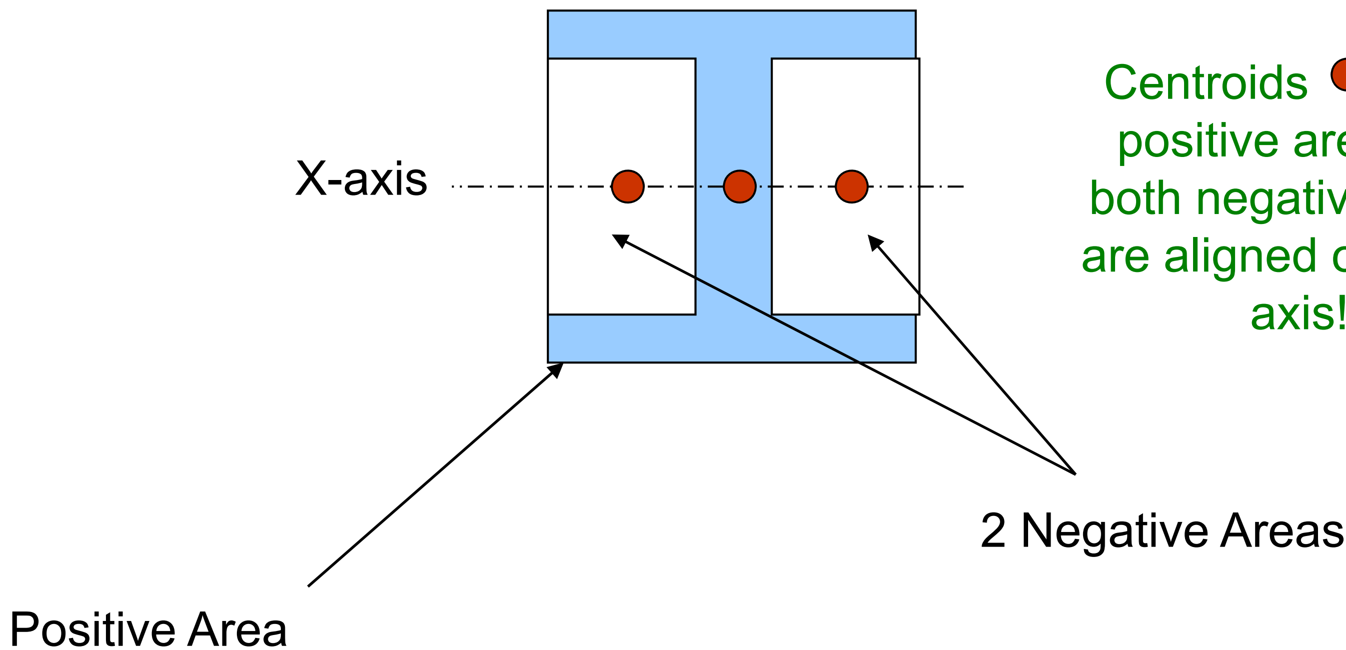
I Beams

- The moment of inertia about the X-axis of an I-beam can be calculated in a similar manner.



I Beams

- Identify the positive and negative areas…



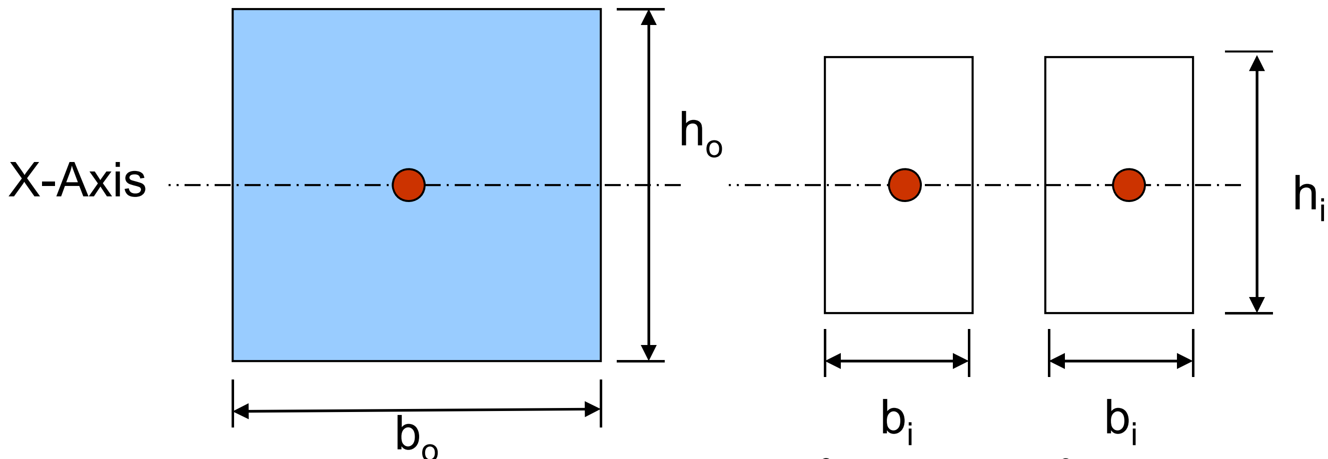
Centroids ● of the positive area and both negative areas are aligned on the x-axis!

$$I_{pos} = \frac{b_o h_o^3}{12} \qquad I_{neg} = \frac{b_i h_i^3}{12}$$

I Beams

- …and calculate the moment of inertia about the X-axis similar to the box beam
- Remember there are two negative areas!

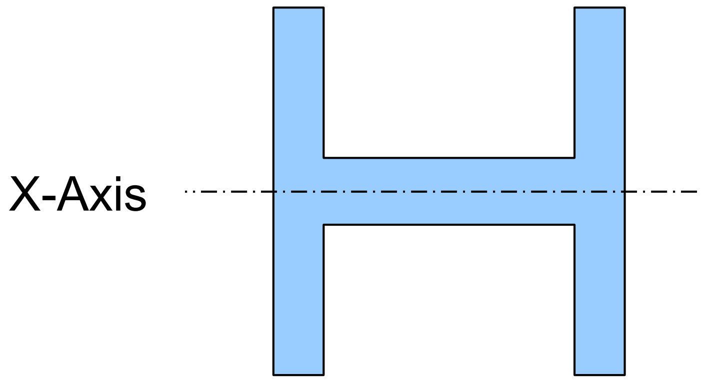
$$I_{total} = I_{pos} - 2 * I_{neg}$$



$$I_{I-beam} = \frac{b_o h_o^3}{12} - \frac{2 b_i h_i^3}{12}$$

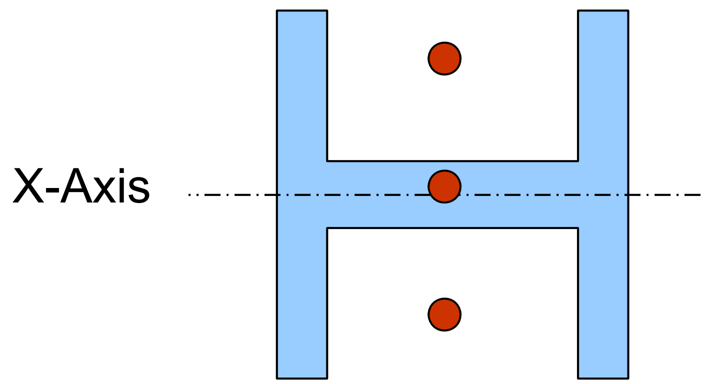
H Beams

- Can we use the “positive-negative area” approach to calculate the Moment of Inertia about the X-axis (I_x) on an H-Beam?



H Beams

- Where are the centroids located?

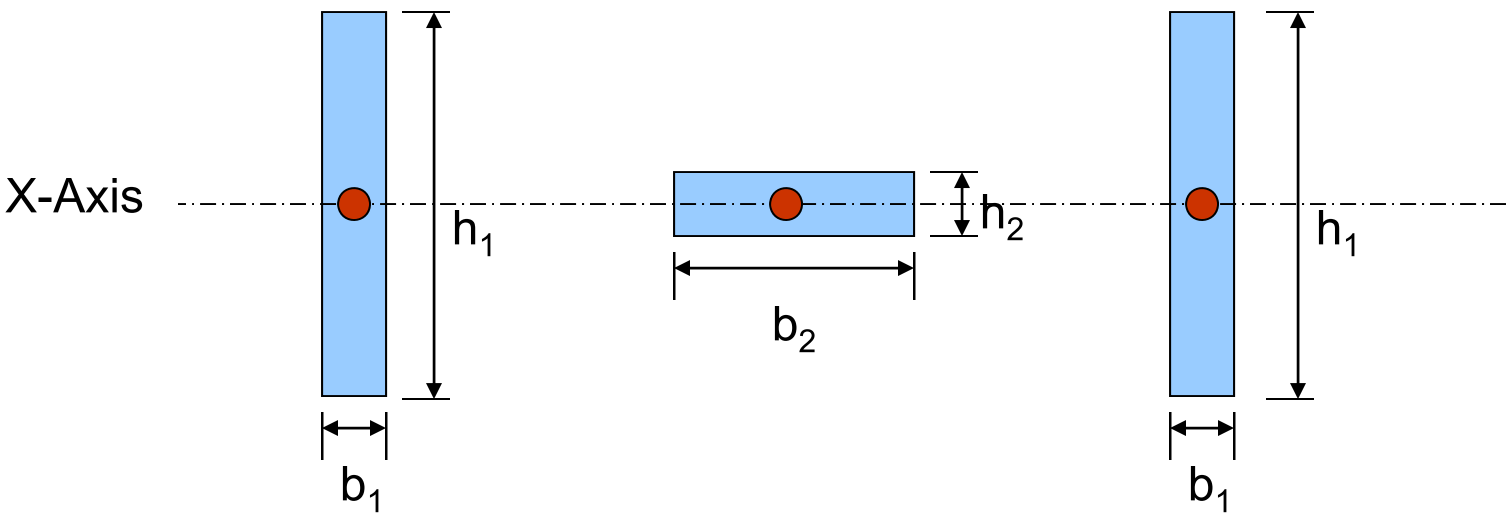


They don’t align on the X-axis. Therefore, we can’t use the “positive-negative approach” to calculate I_x!

We could use it to calculate I_y…but that’s beyond the scope of this class.

H Beams

- We need to use a different approach.
- Divide the H-beam into three positive areas.
- Notice the centroids for all three areas are aligned on the X-axis.



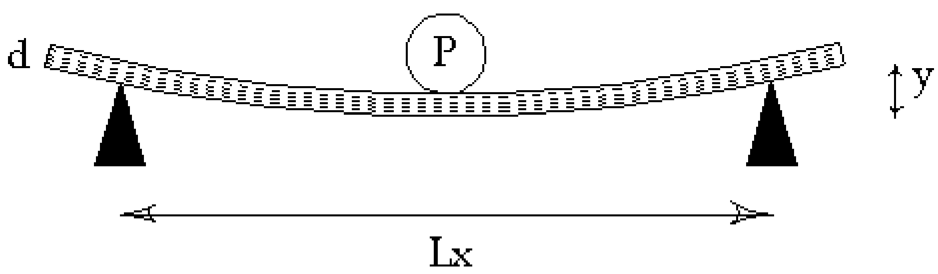
$$I_{H\text{-beam}} = \frac{b_1 h_1^3}{12} + \frac{b_2 h_2^3}{12} + \frac{b_1 h_1^3}{12}$$

OR

$$I_{H\text{-beam}} = \frac{2 b_1 h_1^3}{12} + \frac{b_2 h_2^3}{12}$$

Assignment Requirements

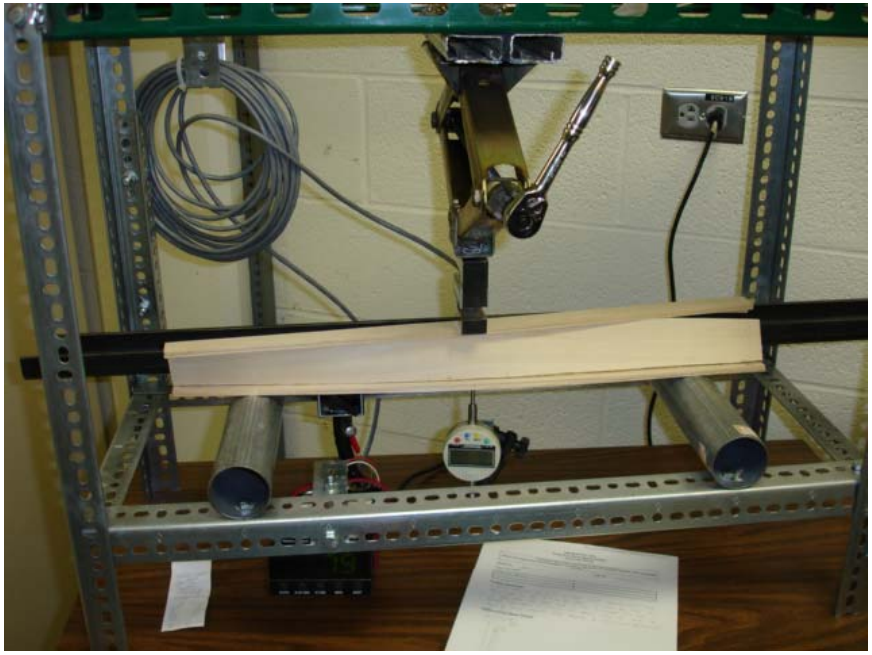
- **Individual**
  - Sketches of 3 beam alternatives
  - **Engineering calculations**
  - Decision matrix
  - Final recommendation to team



- **Team**
  - Evaluate designs proposed by all members
  - Choose the top 3 designs proposed by all members
  - Evaluate the top 3 designs
  - Select the best design
  - Submit a Test Data Sheet
    - Sketch of final design
    - **Engineering calculations**
    - Decision matrix
    - Materials receipt

Test Data Sheet

- Problem statement
- Sketch of final design
- Calculations
- Decision Matrix
- Bill of materials and receipts
- Performance data
  - Design load
  - Volume
  - Weight
  - Moment of Inertia
  - Deflection

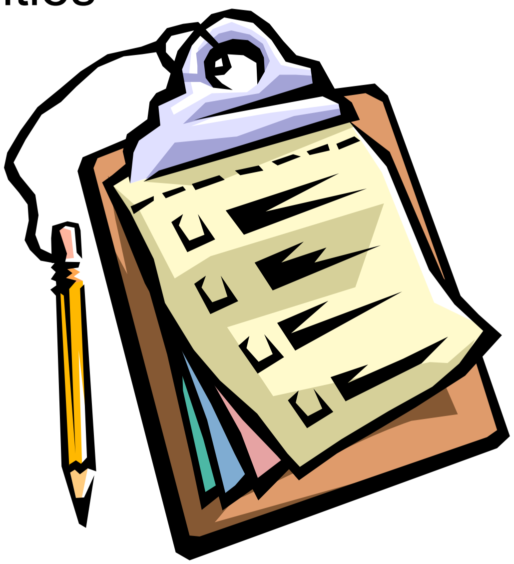


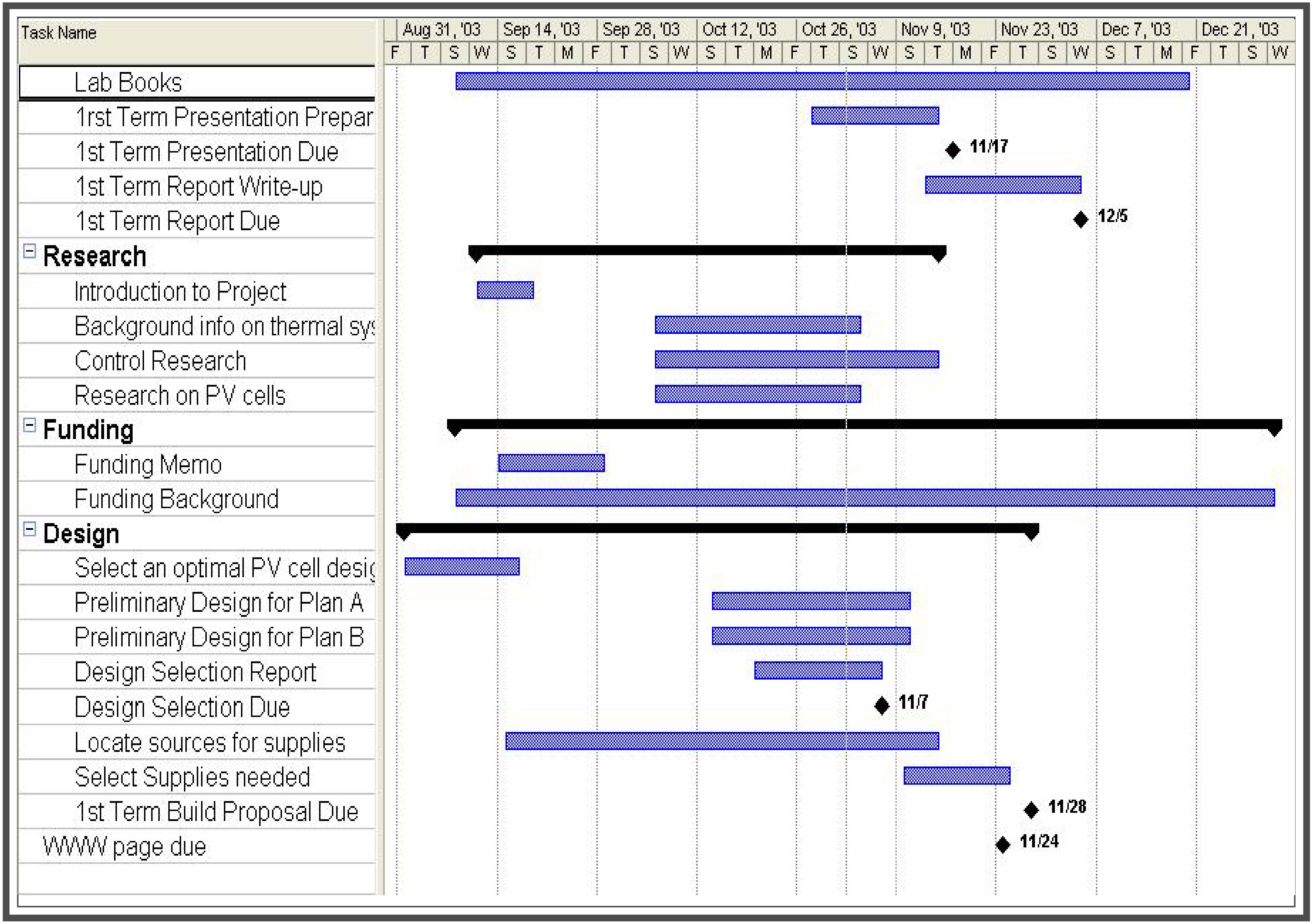
Engineering Presentation

- Agenda
- Problem definition
  - Design Requirements
  - Constraints
  - Assumptions
- **Project Plan**
  - Work Breakdown Structure
  - Schedule
  - Resources
- Research Results
  - Benchmark Investigation
  - Literature Search
- Proposed Design Alternatives
- Alternatives Assessment (Decision Matrix)
- Final Design
- Benefits and Costs of the Final Design
- Expected vs. Actual Costs
- Expected vs. Actual Performance
- Project Plan Results
- Conclusion and Summary

Project Plan

- Start with the 5-step design process
- Develop a work breakdown structure
  - List all tasks/activities
  - Determine priority and order
  - Identify milestone and critical path activities
- Allocate resources
- Create a Gantt chart
  - MS Project
  - Excel
  - Word

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Aug 31, '03
Sep 14, '03
Sep 28, '03
Oct 12, '03
Oct 26, '03
Nov 9, '03
Nov 23, '03
Dec 7, '03
Dec 21, '03
F T S W S T M F T S W S T M F T S W S T M F T S W S T M F T S W
Lab Books
1rst Term Presentation Prepar
1st Term Presentation Due
11/17
1st Term Report Write-up
1st Term Report Due
12/5
Research
Introduction to Project
Background info on thermal sys
Control Research
Research on PV cells
Funding
Funding Memo
Funding Background
Design
Select an optimal PV cell desig
Preliminary Design for Plan A
Preliminary Design for Plan B
Design Selection Report
Design Selection Due
11/7
Locate sources for supplies
Select Supplies needed
1st Term Build Proposal Due
11/28
WWW page due
11/24

For Next Class…

- Read Chapter 8, Introduction to Engineering, pages 227 through 273