DC Analysis of a Common-Emitter Amplifier

The Common-Emitter Amplifier is used to achieve high voltage gain and employs a bi-junction transistor (BJT). A diagram of the common-emitter amplifier is shown in figure 1.

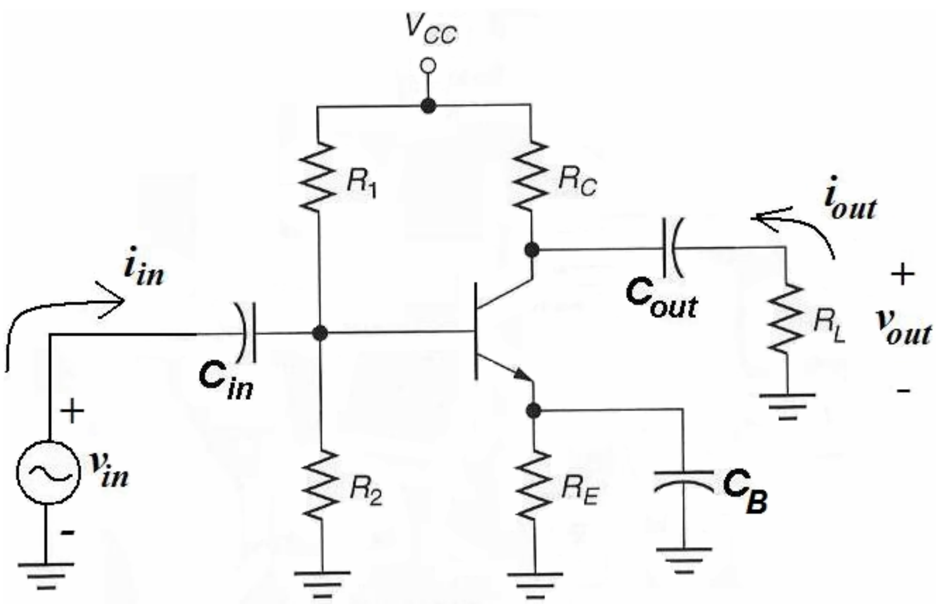

Figure 1. Common emitter (CE) amplifier circuit

The AC voltage v_{in} is provided by an audio source such as a microphone or an MP3 player.

The resistance R_L is the load resistance of the next amplifier stage, or could be the resistance of an audio speaker.

One purpose of C_{in} is to prevent DC current from flowing from V_{cc} to v_{in}, and thus preventing damage to the audio source. Another purpose of C_{in} is to provide an AC short between v_{in} and the base of the transistor, allowing AC current i_{in} to freely flow through the capacitor.

The purpose of C_{out} is to prevent DC current from flowing from V_{cc} to v_{out}. This will prevent damage to the audio speaker or it will prevent DC loading effects on the next amplifier stage. Another purpose of C_{out} is to provide an AC short between the collector of the transistor and the load resistance, allowing AC current i_{out} to freely flow through the capacitor.

DC Analysis.

For DC analysis, we can remove all capacitors, since there is no DC current through them. The DC equivalent circuit is shown in Figure 2.

Parameter definitions:

I_B = DC Base Current
I_C = DC Collector Current
I_E = DC Emitter Current
V_B = DC Base Voltage
V_C = DC Collector Voltage
V_E = DC Emitter Voltage
V_{BE} = DC Base-Emitter Voltage
V_{CC} = DC Supply Voltage
h_{FE} = DC Current Gain $= \frac{I_C}{I_B}$

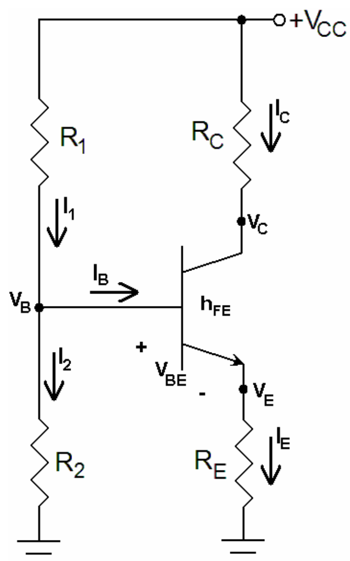

Figure 2. DC equivalent circuit of a common emitter amplifier

<u>About transistor amplifiers</u> When a transistor is operating as an amplifier, the DC current gain (h_{FE}) is a given constant value. The typical values for h_{FE} range from 75 to 200, depending on the type of BJT. For the 2N3904, which is a very commonly used BJT h_{FE} is typically about 150. Another characteristic of a correctly operation transistor amplifier is that the base-emitter voltage (V_{BE}) is about 0.7 volts.

The resistors R1 and R2 form a voltage divider to provide a stable base voltage. One can test to see if the voltage divider is working correctly by removing the transistor and measuring the base voltage. A stable circuit is achieved if the base voltage is about the same whether or not the transistor is in the circuit.

DC analysis:

Given the amplifier parameters:

V_{BE} = DC Base-Emitter Voltage
V_{CC} = DC Supply Voltage
h_{FE} = DC Current Gain
R_1 = First Voltage Divider Resistor
R_2 = Second Voltage Divider Resistor
R_C = Collector Resistor
R_E = Emitter Resistor

Compute the electrical quantities

I_B = DC Base Current
I_C = DC Collector Current
I_E = DC Emitter Current
V_B = DC Base Voltage
V_C = DC Collector Voltage
V_E = DC Emitter Voltage

By Ohm's law, the emitter voltage is $V_E = I_E R_E$ (1)

By KVL, the base voltage is $V_B = V_{BE} + V_E$ (2)

Substituting (2) into (1) gives us $V_B = V_{BE} + I_E R_E$ (3)

Let's assume the transistor is operating correctly with a base-emitter voltage (V_{BE}) is about 0.7 volts. Therefore, the base voltage is:

$$V_B = 0.7 + I_E R_E \tag{4}$$

The transistor currents obey KCL:

$$I_E = I_B + I_C \tag{5}$$

Substituting $I_C = h_{FE} I_B$ into (5) gives us

$$\begin{aligned} I_E &= I_B + h_{FE} I_B \\ &= I_B (h_{FE} + 1) \end{aligned} \tag{6}$$

Substituting (6) into (4) yields

$$\begin{aligned} V_B &= 0.7 + V_E \\ &= 0.7 + I_B (h_{FE} + 1) R_E \end{aligned} \tag{7}$$

Ohm's law for the base voltage through resistor R_2 is:

$$V_B = I_2 R_2 \tag{8}$$

Equating (7) with (8) yields

$$I_2 R_2 = 0.7 + I_B (h_{FE} + 1) R_E \tag{9}$$

By KCL, the base current can be expressed as:

$$I_B = I_1 - I_2 \tag{10}$$

Substituting (11) into (10), yields

$$I_2 R_2 = 0.7 + (I_1 - I_2)(h_{FE} + 1) R_E \tag{11}$$

We can arrange (11) into the form, $aI_1 + bI_2 = c$, as follows:

$$\begin{aligned} I_2 R_2 &= 0.7 + (I_1 - I_2)(h_{FE} + 1) R_E \\ &= 0.7 + (I_1)(h_{FE} + 1) R_E - (I_2)(h_{FE} + 1) R_E \end{aligned} \tag{12}$$

Group current terms of the left side to obtain

$$-I_1 (h_{FE} + 1) R_E + I_2 R_2 + I_2 (h_{FE} + 1) R_E = 0.7 \tag{13}$$

Simplify left side to obtain

$$-I_1 (h_{FE} + 1) R_E + I_2 (R_2 + (h_{FE} + 1) R_E) = 0.7 \tag{14}$$

KVL from V_{CC} through R_1 and R_2 yields

$$I_1 R_1 + I_2 R_2 = V_{CC} \tag{15}$$

Equations (14) and (15) form a system of two equations and two unknowns (I_1 and I_2). These can be formed into a matrix equation:

$$\begin{bmatrix} -(h_{FE} + 1) R_E & R_2 + (h_{FE} + 1) R_E \\ R_1 & R_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.7 \\ V_{CC} \end{bmatrix} \tag{16}$$

The matrix equation can be solved by inverting the resistance matrix:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -(h_{FE} + 1) R_E & R_2 + (h_{FE} + 1) R_E \\ R_1 & R_2 \end{bmatrix}^{-1} \begin{bmatrix} 0.7 \\ V_{CC} \end{bmatrix}$$

A 2x2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

has an inverse of

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example 1. Given the amplifier parameters:

V_{BE} = 0.7 V
V_{CC} = 12 V
h_{FE} = 100
R_1 = 5000
R_2 = 10000
R_C = 2000
R_E = 1000

Compute the electrical quantities

I_B = DC Base Current
I_C = DC Collector Current
I_E = DC Emitter Current
V_B = DC Base Voltage
V_C = DC Collector Voltage
V_E = DC Emitter Voltage
$V_{Divider}$ = DC Voltage at the base of the voltage divider if the transistor is removed.

Solution:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -(h_{FE}+1)R_E & R_2 + (h_{FE}+1)R_E \\ R_1 & R_2 \end{bmatrix}^{-1} \begin{bmatrix} 0.7 \\ V_{CC} \end{bmatrix}$$

Substitute given parameters:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -(100+1)1000 & 5000 + (100+1)1000 \\ 10000 & 5000 \end{bmatrix}^{-1} \begin{bmatrix} 0.7 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} -(101)1000 & 5000 + (101)1000 \\ 10000 & 5000 \end{bmatrix}^{-1} \begin{bmatrix} 0.7 \\ 12 \end{bmatrix} \qquad \text{(P1.1)}$$

$$= \begin{bmatrix} -101000 & 106000 \\ 10000 & 5000 \end{bmatrix}^{-1} \begin{bmatrix} 0.7 \\ 12 \end{bmatrix}$$

The 2x2 resistor matrix

$$R = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -101000 & 106000 \\ 10000 & 5000 \end{bmatrix}$$

has an inverse of

$$R^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{(-101000)(5000)-(106000)(10000)} \begin{bmatrix} 5000 & -106000 \\ -10000 & -101000 \end{bmatrix} \qquad \text{(P1.2)}$$

$$= \frac{1}{(-505000000)-(1060000000)} \begin{bmatrix} 5000 & -106000 \\ -10000 & -101000 \end{bmatrix}$$

$$= \frac{1}{-1.565e9} \begin{bmatrix} 5000 & -106000 \\ -10000 & -101000 \end{bmatrix}$$

Substituting (P1.2) into (P1.1) yields

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -101000 & 106000 \\ 10000 & 5000 \end{bmatrix}^{-1} \begin{bmatrix} 0.7 \\ 12 \end{bmatrix}$$

$$= \frac{1}{-1.565e9} \begin{bmatrix} 5000 & -106000 \\ -10000 & -101000 \end{bmatrix} \begin{bmatrix} 0.7 \\ 12 \end{bmatrix}$$

$$= \frac{1}{-1.565e9} \begin{bmatrix} (5000)(0.7) + (-106000)(12) \\ (-10000)(0.7) + (-101000)(12) \end{bmatrix}$$

$$= \frac{1}{-1.565e9} \begin{bmatrix} (3500) + (-1272000) \\ (-7000) + (-1212000) \end{bmatrix}$$

$$= \frac{1}{-1.565e9} \begin{bmatrix} -1268500 \\ -1219000 \end{bmatrix}$$

$$= \begin{bmatrix} -1268500 / -1.565e9 \\ -1219000 / -1.565e9 \end{bmatrix} = \begin{bmatrix} 810.543 \\ 778.914 \end{bmatrix} \times 10^{-6}$$

Therefore, we have the voltage divider currents I_1 = 810.543 μA and I_2 = 778.914 μA.

The base current is computed from (10):

$$I_B = I_1 - I_2 = 810.543 - 778.914 = 31.629\ \mu A$$

The collector current is computed from the base current:

$$I_C = h_{FE} I_B = (100)(31.629\ \mu A)$$
$$= (3162.9\ \mu A) = 3.163\, mA$$

The emitter current is computed from KCL (5):

$$I_E = I_B + I_C = 31.629\ \mu A + 3162.9\ \mu A = 3195\ \mu A = 3.195\, mA$$

The emitter voltage is computed from Ohms's Law (1):

$$V_E = I_E R_E = (3.195\, mA)(1000\, \Omega) = 3195\, mV = 3.195\, V$$

The base voltage is computed from KVL (2):

$$V_B = V_{BE} + V_E = 0.7 + 3.195 = 3.895 V$$

The collector voltage is computed from KVL:

$$V_C = V_{CC} - I_C R_C = 12V - (3.163\, mA)(2000\, \Omega)$$
$$= 12V - 6326\, mV = 12V - 6.326V = 5.674V$$

If one removes the transistor, the voltage at the base of the voltage divider is

$$V_{Divider} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{5000}{10000 + 5000}\right)(12V)$$
$$= \left(\frac{5000}{15000}\right)(12V) = \left(\frac{1}{3}\right)(12V) = 4V$$

This is a good result, because it compares favorably with the transistor base voltage of 3.895 volts. This shows that the transistor amplifier will be stable.

The MATLAB program shown in figure 4 implements the Matrix equation (16) and some other equations above to compute all the transistor currents and voltages, given the circuit parameters of a common-emitter amplifier. A sample execution is shown in figure 3. Notice that the results given in figure 3 match the solution provided above for Example 1.

```
********** Common Emitter Amplifier  ***********
Given Parameters
****************
Resistor R1           = 10000 ohms
Resistor R2           = 5000 ohms
Collector Resistor    = 2000 ohms
Emitter Resistor      = 1000 ohms
************************************************

***********  DC Analysis  **************
Base Voltage          = 3.89 volts
Unloaded Base Voltage = 4.00 volts
Collector Voltage     = 5.67 volts
Emitter Voltage       = 3.19 volts
Base Current          = 31.63 uA
Collector Current     = 3.16 mA
Emitter Current       = 3.19 mA
******************************************
```

Figure 3. Sample MATLAB program execution

```
% Common Emitter Amplifier with voltage divider bias
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%           Input parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Re = 1000;  % emitter resistance in ohms
R1 = 10000; % voltage divider resistance R1 in ohms
R2 =  5000; % voltage divider resistance R2 in ohms
hFE = 100;  % DC current gain
Rc = 2000;  % collector resistance in ohms
Vcc = 12;   % DC power supply voltage in volts

fprintf('********** Common Emitter Amplifier  ***********\n');
fprintf('               Given Parameters        \n');
fprintf('               ****************\n')
fprintf('  Resistor R1            = %d ohms\n', R1);
fprintf('  Resistor R2            = %d ohms\n', R2);
fprintf('  Collector Resistor     = %d ohms\n', Rc);
fprintf('  Emitter Resistor       = %d ohms\n', Re);
fprintf('************************************************\n');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%              DC Analysis
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% set up matrix equation
R = [-(hFE+1)*Re    R2+(hFE+1)*Re;
    R1      R2];
V = [0.7
    Vcc];

% Solve matrix equation
I = inv(R)*V  ;
I1 = I(1); % current in resistor R1
I2 = I(2); % current in resistor R2

% Compute transistor currents and voltages
Ib = I1 - I2; % base current
Ic = b*Ib;  % collector current
Ie = Ib+Ic; % emitter current
Ve = Ie*Re; % emitter voltage
Vb =Ve+0.7; % base voltage
Vc = Vcc - Ic*Rc; % collector voltage
VbUnloaded = Vcc*R2/(R1+R2); % base voltage with BJT removed


% results
fprintf('\n*******  DC Analysis Results **********\n');
fprintf('  Base Voltage           = %.2f volts\n', Vb);
fprintf('  Unloaded Base Voltage = %.2f volts\n', VbUnloaded);
fprintf('  Collector Voltage      = %.2f volts\n', Vc);
fprintf('  Emitter Voltage        = %.2f volts\n', Ve);
fprintf('  Base Current           = %.2f uA\n', Ib*1e6);
fprintf('  Collector Current      = %.2f mA\n', Ic*1000);
fprintf('  Emitter Current        = %.2f mA\n', Ie*1000);
fprintf('*****************************************\n');
```

Figure 4. MATLAB program for the DC analysis of a common emitter amplifier