Sleep and the Shift Worker: A Mathematical Biology Approach to an Age-Old Problem

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History of Shift Work

- Always present to some degree: soldiers, sailors, physicians
- Late 1800's/Early 1900's: industries considered extending to night work
  - 1883 Edison invents electric light bulb
  - 1914 Ford brings shift work to auto industry
  - 1914 - 1918 World War I, 1939 - 1945 World War II
- Continued demand in Mid-Late 1900's in industry and service related occupations
- Present transition towards "flex" schedules: May 2004 statistics [1]
  - Over 27 million full-time workers with flexible schedules (27.5% of full time work force; 14.8% of which were shift workers)
  - 44.7% of management, business and financial operations workers;
  - 29.5% of sales and office workers;
  - 17.6% of construction and maintenance workers;
  - 14.3% of production, transportation, and material moving workers.

Circadian Rhythms

What are circadian rhythms?

- Biological rhythms that repeat approximately every 24 hours.
- Examples:
  Hormone levels (cortisol, melotonin, thyroid-stimulating hormone), Body temperature, Sleep/wake patterns, Alertness

Why are circadian rhythms studied?

- Natural tie to alertness levels
- Uses of chronobiological research:
  Treating sleep disorders, Adaption to jet-lag, Adaption of astronauts to 'round-the-clock work, Design of rotating shift work schedules

Shift Work

What constitutes shift work?

- Any work not occurring between the hours of 6 a.m. and 6 p.m.
- Characterizing features of shift work:
  - Permanent or rotating shifts
  - Length of a single shift
  - Number/Pattern of days worked in a week
  - Time of day the shift occurs
  - Rotation pattern of non-permanent shifts

Shift Work Problems

What are some of the problems associated with shift work?

- Interference with social and domestic life
- Decreases in health and wellness
  - Gastrointestinal and cardiovascular disorders
  - Increased risk of breast cancer
  - Disturbed sleep and fatigue
  - Depression
- Low-productivity
- On-the-job accidents
  (Three Mile Island, Bhopal, Chernobyl, Exxon-Valdez accidents
  all occurred between midnight and 4 a.m.)

Linking Circadian Rhythms, Shift Work, and Mathematics

Why is the study of circadian rhythms relevant to shift work?

- Some of these problems can be traced back to physiological disturbances in circadian rhythms.
- Recommendations for designing shift schedules that minimize adverse effects of shift work on human health and performance.

How does mathematics fit in?

- Circadian criteria can be used to help develop shift scheduling algorithms.
- Mathematical models of circadian rhythms can help shed light on the dynamics of circadian rhythms.

Using Circadian Criteria to Design Shift Schedules

Chronobiological Research

- Suggestions for designing shift schedules that adhere to circadian principles regarding
  - Speed of rotation
  - Direction of rotation
  - Duration of a single shift
  - Start time of each shift
  - Distribution of days off
- Differing opinions on characteristics of preferred shift work schedules
  - Czeisler: slow forward rotating shifts; same shift for several weeks [2]
  - Knauth: rapid forward rotating shifts; several shifts in same week; [3]
  - Turek: no "optimal" direction [4]
  - Monk: no "optimal" speed [5]

Using Circadian Criteria to Design Shift Schedules

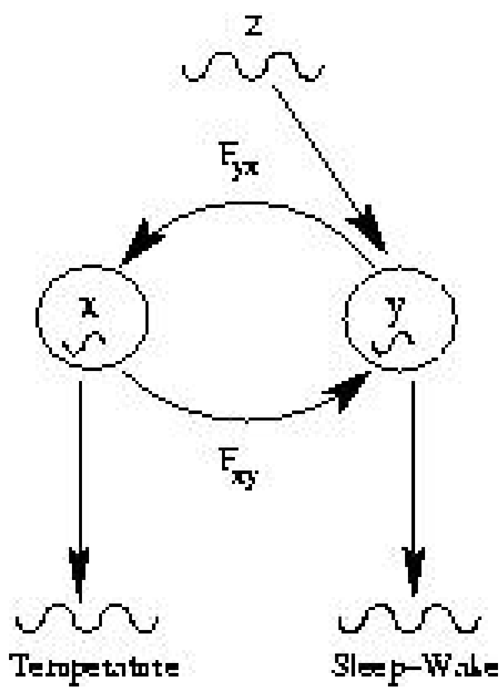
Designing and Evaluating Shift Schedules

- Designing schedules to meet operational demands as well as ergonomic and circadian criteria:
  - Kostreva, Genvier, and Jennings [6]
  - Nachreiner *et al.* [7]
  - Chen and Yeung [8]
  - Kostreva and Genvier [9]
- Evaluating shift schedules based on circadian criteria
  - Gissel and Knauth [10]
  - Pilcher, Lambert and Huffcutt [11]
  - Saunders [12]

Mathematical Models of Circadian Rhythms

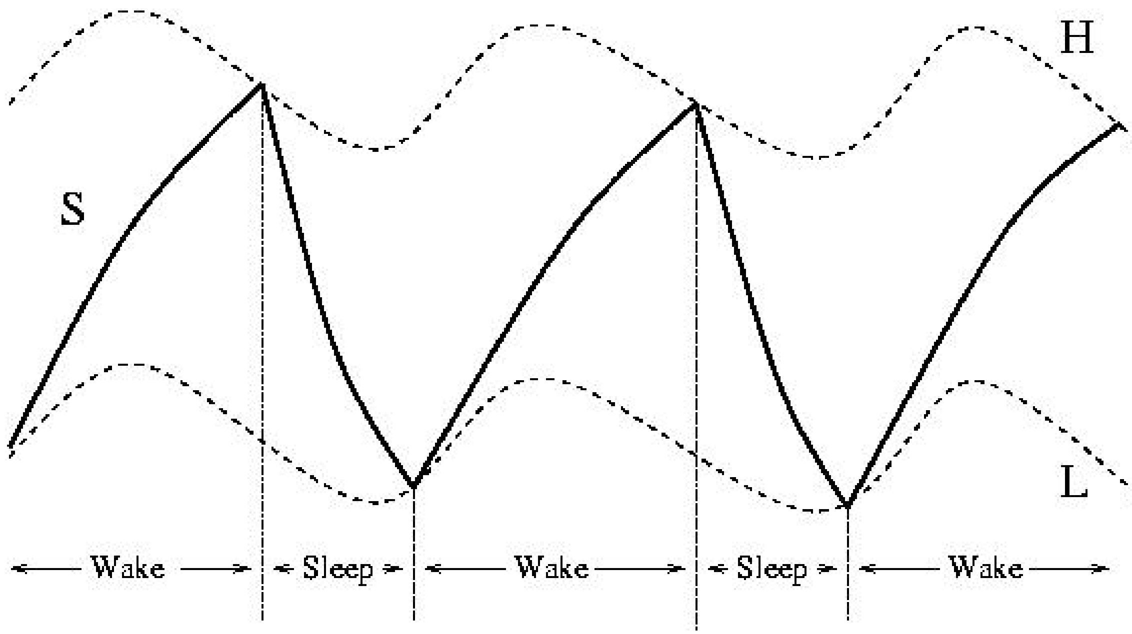
A Few Schools of Models:

1. Multi-Oscillator Models



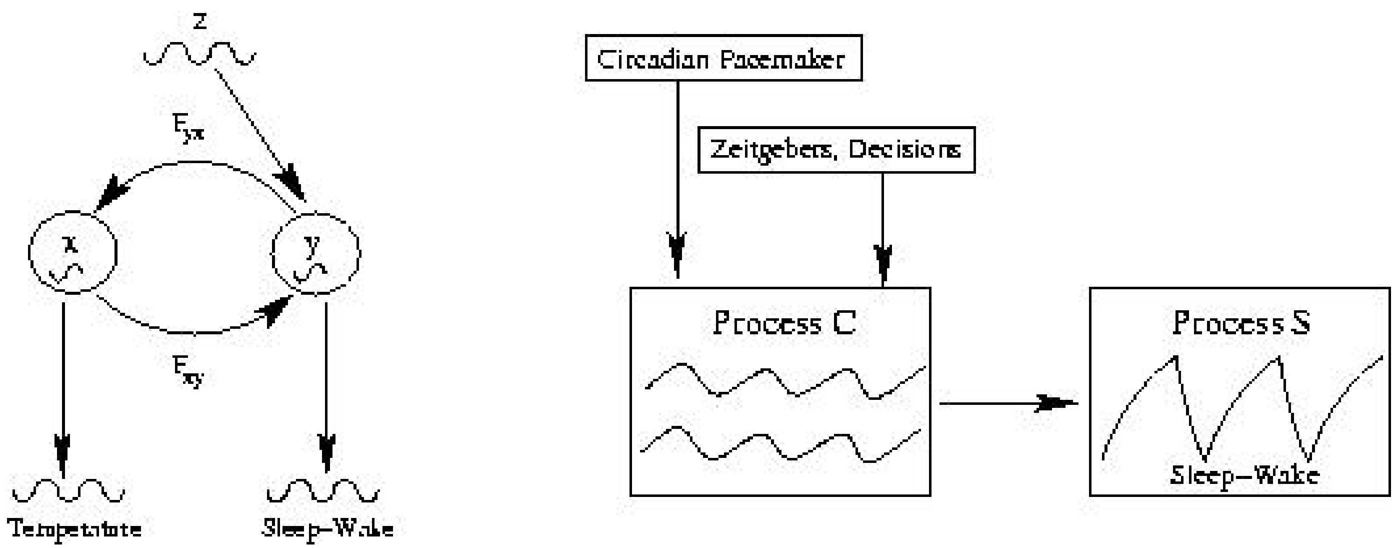
Two-Oscillator Model of Kronauer et al. [13]

2. Multi-Process Models



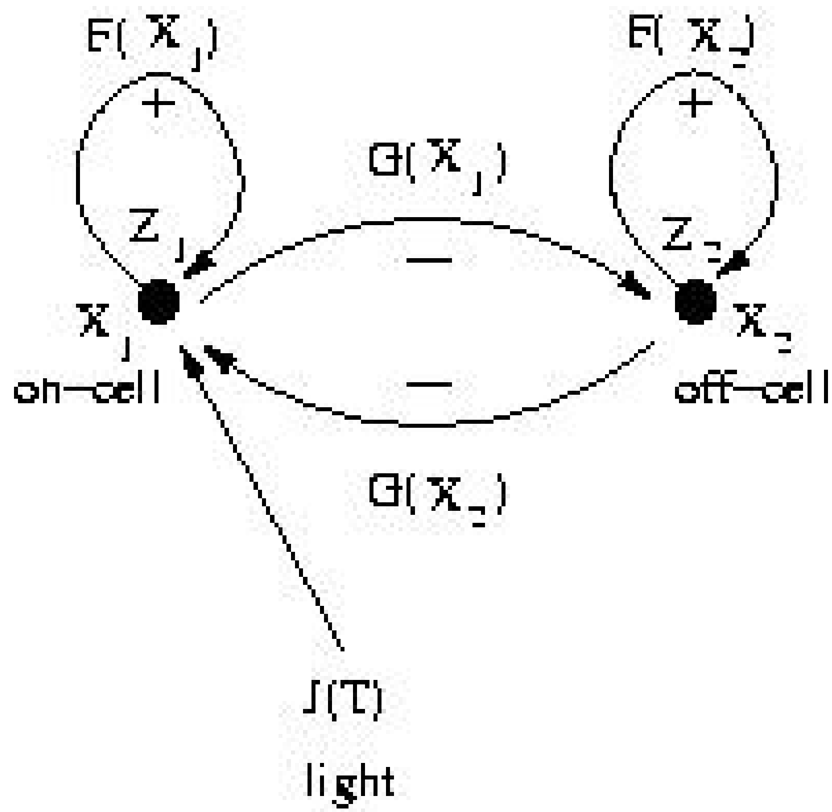
Two-Process Model of Daan, Beersma, and Borbély [14]

Comparing Models



Comparing the Two-Oscillator and Two-Process Model

3. Neural Pacemakers



Gated Neural Pacemaker of Carpenter and Grossberg

Combining Previous Types of Research

Objectives:

- To develop a model for circadian rhythms of a laborer working specified shift schedules,

- To develop a method for quantifying the degree to which a given shift schedule disrupts circadian rhythms by comparing work-related rhythms to natural benchmark rhythms

- To use this method to:
  - Evaluate the circadian compatibility of a given shift schedule
  - Develop general shift work schedules that least disturb the shift worker's natural circadian rhythms

Kronauer's Circadian Rhythms Model [16]

$$\left(\frac{24}{2\pi}\right)^2 \ddot{x} + \mu(-1 + 4x^2)\left(\frac{24}{2\pi}\right)\dot{x} + \left(\frac{24}{\tau_x}\right)^2 x \;=\; \left(\frac{24}{2\pi}\right)\dot{B} \quad (1)$$

where

$$B = (1 - mx)CI^{1/3} \quad (2)$$

x represents the temperature oscillator,

B represents the "perceived" brightness,

I represents the physical intensity of light,

μ represents the internal "stiffness" of the x oscillator,

τ_x represents the intrinsic period of the x oscillator,

m is a modulation index,

C is a constant of proportionality, and

$\frac{24}{2\Pi}$ is the time parameter converting one unit of time to one hour.

Adapting the Model to Shift Work

- Through Consequential Changes to Light Intensity Function, $I(t)$
- Through Incorporation of a Shift Work Zeitgeber Function, $z(t)$

$$z(t) = \begin{cases} k & t^* \leq t \leq t^* + 8 \\ 0 & otherwise \end{cases} \tag{3}$$

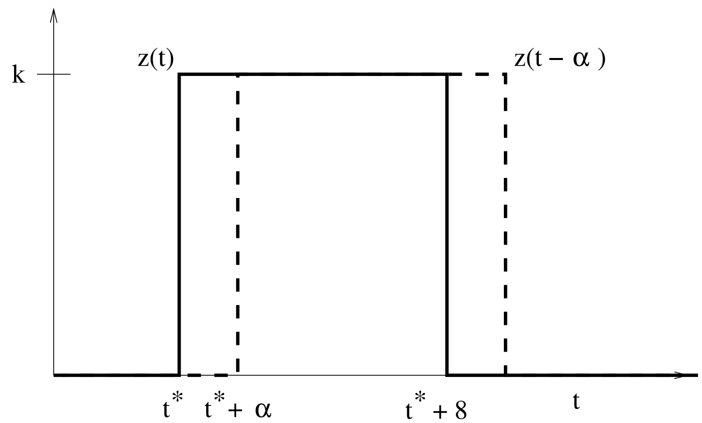
where k is a constant and t^* is the starting time of the shift to be worked. Then the model for shift work modified circadian rhythms is taken to be

$$\left(\frac{24}{2\pi}\right)^2 \ddot{x} + \mu(-1 + 4x^2)\left(\frac{24}{2\pi}\right)\dot{x} + \left(\frac{24}{\tau_x}\right)^2 x \;=\; \left(\frac{24}{2\pi}\right)\dot{B} + z(t) \tag{4}$$

Parameterization of the Problem

Adjusting the Shift Work Sleep-Wake Equation and Zeitgeber

$$\left(\frac{24}{2\pi}\right)^2 \ddot{x} + \mu(-1+4x^2)\left(\frac{24}{2\pi}\right)\dot{x} + \left(\frac{24}{\tau_x}\right)^2 x \;=\; \left(\frac{24}{2\pi}\right)\dot{B} + z(t-\alpha) \quad (5)$$



The Parameterized Shift Work Function $z(t-\alpha)$

Determining an Optimal Shift Schedule

Objective:

Given a set schedule of days to work, determine the optimal time of the day, α^*, in which to start that work schedule. In other words, find the value of α that minimizes the error associated with such a shift schedules sleep-wake rhythms, i.e.

$$\min_{0 \le \alpha \le 24} \sqrt{\sum_{i=1}^{n} (x_f(t_i) - x_{w(\alpha)}(t_i))^2}$$

Assumptions:

- The shift schedule consists of five days of work and two days off of work.
- Each shift lasts exactly 8 hours.
- Time $t = 0$ corresponds to 12 a.m. Monday morning.
- A shift can start at *any* time during the day.

Numerical Methods and Solution Procedure

- Define $F(\alpha) = \sqrt{\sum_{i=1}^{n}(x_f(t_i) - x_{w(\alpha)}(t_i))^2}$.

- Evaluate F for the values $\alpha = 1, 2, \cdots, 24$.
  Note, each evaluation of F requires solving the system of differential equations with MATLAB's ode solver.

- Determine an interval $[a, b]$ over which $F(\alpha)$ is unimodal and attains its minimum.

- Use Golden Section routine to find the value of α that minimizes F.

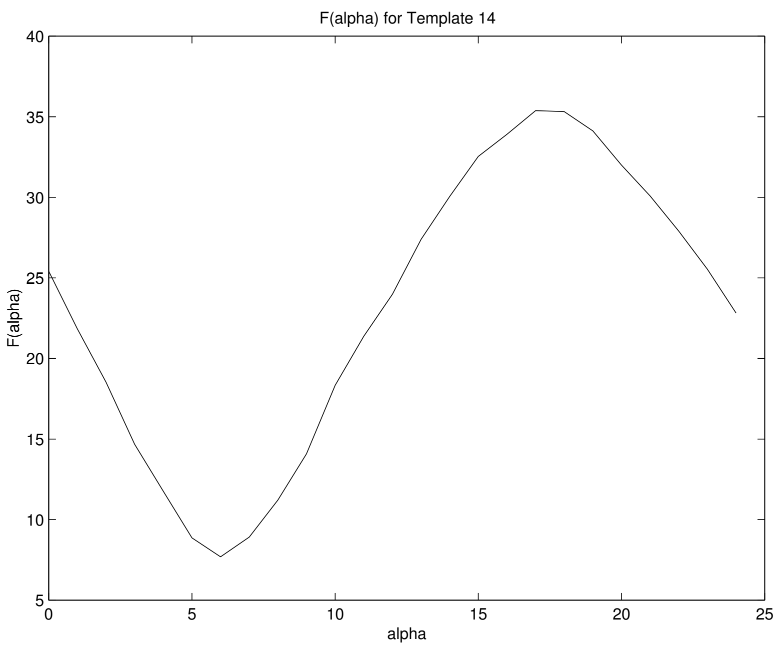
Twenty-One Possible Single-Week Schedules

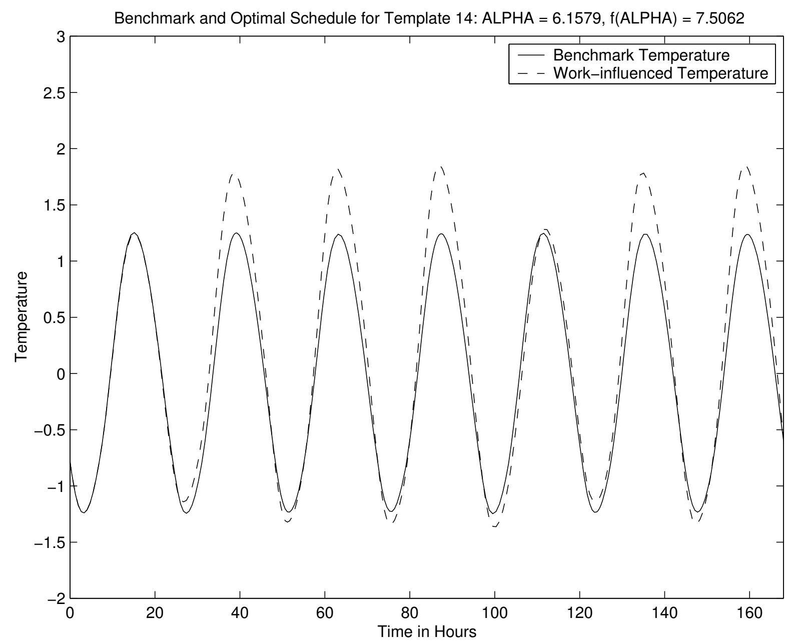
Schedule #			Days	Worked			
1	M	Tu	W	Th	F		
2	M	Tu	W	Th		Sa	
3	M	Tu	W	Th			Su
4	M	Tu	W		F	Sa	
5	M	Tu	W			Sa	Su
6	M	Tu		Th	F	Sa	
7	M	Tu			F	Sa	Su
8	M		W	Th	F	Sa	
9	M			Th	F	Sa	Su
10		Tu	W	Th	F	Sa	
11			W	Th	F	Sa	Su
12		Tu		Th	F	Sa	Su
13		Tu	W		F	Sa	Su
14		Tu	W	Th		Sa	Su
15		Tu	W	Th	F		Su
16	M		W		F	Sa	Su
17	M		W	Th		Sa	Su
18	M		W	Th	F		Su
19	M	Tu		Th		Sa	Su
20	M	Tu		Th	F		Su
21	M	Tu	W		F		Su

First Week Investigations: Sample Results

Smallest Optimal $F(\alpha)$ Value:
Schedule 14 with $F(\alpha) = 7.50618$

M	Tu	W	Th	F	Sa	Su





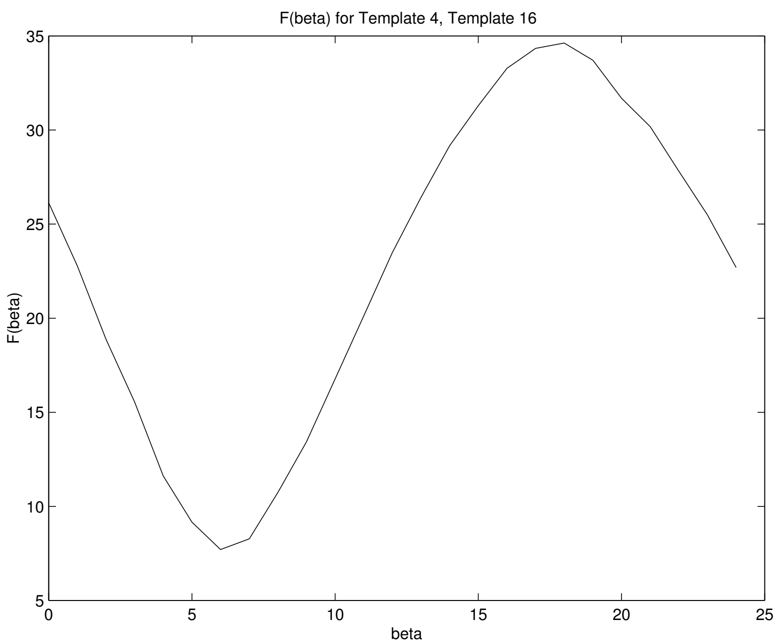
$F(\alpha)$ Graph and Optimal $x_w(t)$ Rhythm for Schedule 14

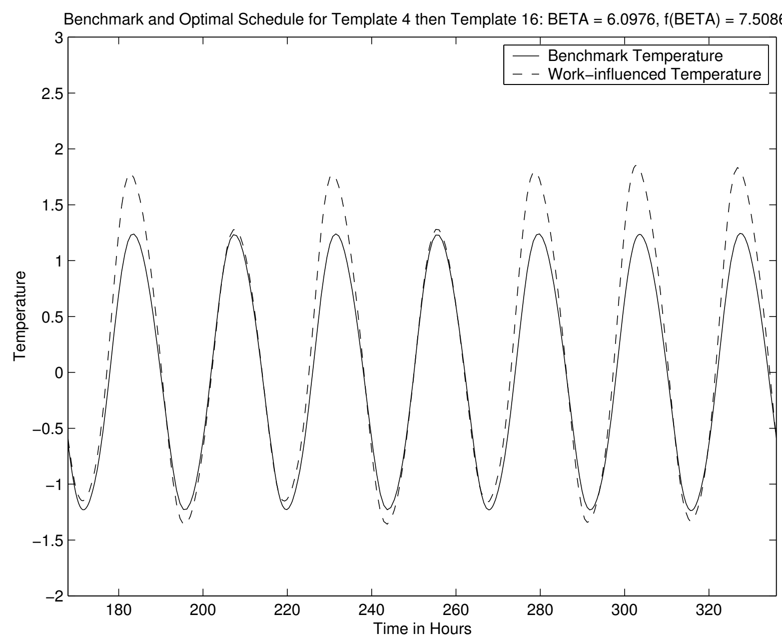
Second Week Investigations: Sample Results

Smallest Optimal $F(\alpha) + F(\beta)$ Value:

Schedule 4 to Schedule 16 with $F(\alpha) + F(\beta) = 15.053$

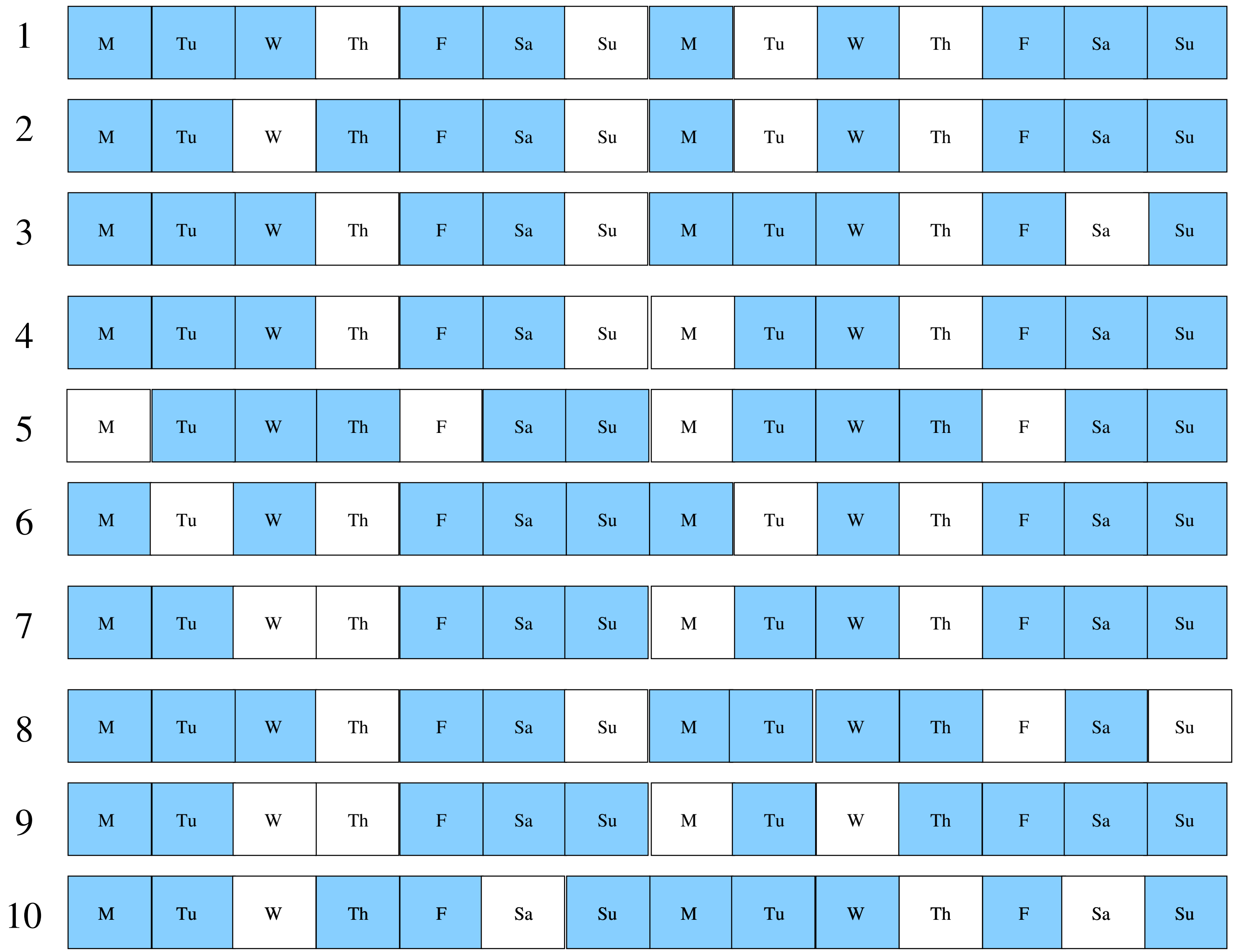
M	Tu	W	Th	F	Sa	Su	M	Tu	W	Th	F	Sa	Su





$F(\beta)$ Graph and Optimal $x_w(t)$ Rhythm for Schedule 16

Top Ten Schedules



Top Twenty-One Two-Week Schedules

M	**Tu**	**W**	Th	**F**	**Sa**	Su	**M**	Tu	**W**	Th	**F**	**Sa**	**Su**
M	**Tu**	W	**Th**	**F**	**Sa**	Su	**M**	Tu	**W**	Th	**F**	**Sa**	**Su**
M	**Tu**	**W**	Th	**F**	**Sa**	Su	**M**	**Tu**	**W**	Th	**F**	Sa	**Su**
M	**Tu**	**W**	Th	**F**	**Sa**	Su	M	**Tu**	**W**	Th	**F**	**Sa**	**Su**
M	**Tu**	**W**	**Th**	F	**Sa**	**Su**	M	**Tu**	**W**	**Th**	F	**Sa**	**Su**
M	Tu	**W**	Th	**F**	**Sa**	**Su**	**M**	Tu	**W**	Th	**F**	**Sa**	**Su**
M	**Tu**	W	Th	**F**	**Sa**	**Su**	M	**Tu**	**W**	Th	**F**	**Sa**	**Su**
M	**Tu**	**W**	Th	**F**	**Sa**	Su	**M**	**Tu**	**W**	**Th**	F	**Sa**	Su
M	**Tu**	W	Th	**F**	**Sa**	**Su**	M	**Tu**	W	**Th**	**F**	**Sa**	**Su**
M	**Tu**	W	**Th**	**F**	Sa	**Su**	**M**	**Tu**	**W**	Th	**F**	Sa	**Su**
M	**Tu**	**W**	Th	F	**Sa**	**Su**	**M**	**Tu**	**W**	Th	**F**	Sa	**Su**
M	**Tu**	**W**	**Th**	F	**Sa**	**Su**	M	**Tu**	**W**	Th	**F**	**Sa**	**Su**
M	**Tu**	**W**	**Th**	F	**Sa**	**Su**	M	**Tu**	W	**Th**	**F**	**Sa**	**Su**
M	**Tu**	W	**Th**	**F**	**Sa**	Su	**M**	**Tu**	W	**Th**	F	**Sa**	**Su**
M	**Tu**	W	**Th**	**F**	**Sa**	Su	**M**	**Tu**	**W**	Th	**F**	Sa	**Su**
M	**Tu**	**W**	Th	**F**	**Sa**	**Su**	M	**Tu**	**W**	Th	**F**	**Sa**	**Su**
M	**Tu**	**W**	**Th**	F	Sa	**Su**	**M**	Tu	W	**Th**	**F**	**Sa**	**Su**
M	**Tu**	**W**	**Th**	**F**	Sa	Su	M	**Tu**	**W**	**Th**	F	**Sa**	**Su**
M	**Tu**	**W**	Th	**F**	**Sa**	Su	**M**	Tu	**W**	**Th**	F	**Sa**	**Su**
M	**Tu**	**W**	**Th**	F	**Sa**	**Su**	**M**	Tu	**W**	**Th**	F	**Sa**	**Su**
M	**Tu**	**W**	Th	**F**	**Sa**	**Su**	M	**Tu**	**W**	**Th**	**F**	Sa	**Su**

Observations and Conclusions

In terms of observations regarding optimal two week shift work schedules:

- Better schedules have:
  - Fewer days off in a row
  - Fewer days worked in a row
  - One or two Thursdays off
- The best shift schedules involve work that starts between the hours of 5:45 a.m. and 6:30 a.m.
- There is no consistently good or poor choice of weekly schedules, it all depends on their combinations.
- The typical Monday through Friday two week schedule is one of the lower caliber performers (ranked 355 out of 441 schedules).

Observations and Conclusions

In terms of statistical analysis of data generated from these simulations:

- Later starting times for the first week of work correspond to decreases in the difference between weekly starting times.

- If the first week of work starts before 6 a.m., the second week of work tends to start later, where as if the first week of work starts after 6 a.m., the second week of work tends to start earlier.

- As the maximum span of consecutive days worked increases, the objective function increases, making these schedules less desirable.

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