MthSc Honors 206 Section 1 – Maple Project 2
Due: Friday, March 1, 2002

'Strike One???'

Let α and β denote the 2^{nd} and 5^{th} integers in your social security number.

[Adapted from 'Calculus using Hewlett-Packard Graphing Calculators,' 4th ed., by Edwards and Pinney, Prentice-Hall, pg 81 - 82 by Dr. Vincent J. Ervin.]

Suppose that a pitcher throws a ball toward home plate (60 ft. away) and gives it a spin of S revolutions per second counter-clockwise (as viewed from above) about a vertical axis through the center of the ball. The spin is described by the *spin vector* $\mathbf{S}$ that points along the axis of revolution in the right-handed direction and has length S.

From studies in aerodynamics, it is well know that this spin causes a difference in air pressure on the sides of the ball toward and away from the spin. This pressure difference results in a *spin acceleration* of the ball

$$\mathbf{a}_S = c\,\mathbf{S} \times \mathbf{v},$$

where c is an empirical constant, and $\mathbf{v}$ is the velocity of the ball. The total acceleration of the ball is then

$$\mathbf{a} = c\,\mathbf{S} \times \mathbf{v} - g\,\mathbf{k}$$

where $g \approx 32$ ft/sec^2 is the gravitational acceleration. [The effects of air resistance will be ignored.]

With the spin vector $\mathbf{S} = S\mathbf{k}$, and the velocity vector $\mathbf{v} = v_x\,\mathbf{i} + v_y\,\mathbf{j} + v_z\,\mathbf{k}$ it is straight forward to show that

$$\mathbf{S} \times \mathbf{v} = -Sv_y\,\mathbf{i} + Sv_x\,\mathbf{j}.$$

For a ball pitched parallel to the x-axis, v_x is much larger than v_y, so assume that $v_y \approx 0$. Hence, the acceleration vector of the ball is given by

$$\mathbf{a} = cSv_x\,\mathbf{j} - g\,\mathbf{k}.$$

Now, assume that the x-axis passes through the middle of 'home plate' and the pitcher's rubber, the point directly below the point from which we will assume the ball is released.

Suppose that the pitcher throws the ball from the initial position of $x_0 = y_0 = 0, z_0 = 5$ (ft.), aimed at a point $2.\alpha$ feet to his right of the front of home plate and $6.\beta$ feet above. We will assume home plate is 1 ft. square. Assume also that the initial speed of release is 12α ft/sec, (120 ft/sec $\approx$ 82 mph), $S = 40$ rev/sec, and $c = 0.005$ ft/sec^2 per feet/second of velocity and revolutions/second of spin.

Questions:

1. If we define 'the flight time' of the ball to be the time taken from being released by the pitcher to crossing the plate determined by the back of home plate, determine the flight time.
2. Plot the path of the ball (using the spacecurve command).
3. Determine how far the ball moves in the 'x' direction during the last 0.025 seconds.
4. Determine how far the ball moves to the right during the last 0.025 seconds.
5. Determine how far the ball 'drops' during the last 0.025 seconds.
6. Determine if the ball will pass through the 'strike zone'. (Clearly state your assumptions concerning the 'strike zone'.)
7. Plot the curvature of the ball's path.
8. For the direction and spin on the ball fixed, determine the range of pitching speeds (i.e. initial speeds of release) which will produce a strike.