# MATH 370 - Probability \& Statistics I Probability Distributions, Means, and Standard Deviations for Discrete Random Variables 

PLINKO!(The Price Is Right)
Game Description: A contestant is shown four small prizes. For each prize, he or she sees two digits, but only one is right. Either the first digit shown is the correct first digit or the last digit shown is the correct last digit. For each small item guessed correctly, the contestant wins a chip. Bob gives the contestant a free chip just for playing the game, so five chips can be earned. The contestant climbs to the top of the Plinko board (see picture below) and drops the chips one at a time. The pegs send the chip bouncing all over the board until they land in slots representing money amounts at the bottom. The slots are, from left to right; $\$ 100, \$ 500, \$ 1000, \$ 0, \$ 10000, \$ 0, \$ 1000, \$ 500, \$ 100$. After all the chips have dropped, the contestant wins whatever money he or she has earned to that point.


The Plinko board has 12 levels. If you start from Slot 5, for example, you need 6 left bounces and 6 right bounces to hit the $\$ 10000$ slot at the bottom.

## Simulating Dropping the Plinko Chip

1. You can simulate one "drop" with the random integer function by using randInt ( $0,1,12$ ) [STO]-> $\mathrm{L}_{1}$ on the TI-83. (Note: randInt ( is found under the MATH $\rightarrow$ PRB menu, option 5.) If each 0 counts as a left bounce and each 1 as a right bounce, the calculator creates 12 values randomly of either 1 or 0 and stores them in List 1 .
2. Tally and record the total number of right bounces. This can be recorded as sum(L1) from the home screen (note the sum() function is found under the LIST $\rightarrow$ MATH menu, option number 5).
3. To find your location at the bottom of the Plinko board, use the conversion table below (which accounts for reflections off the wall). The table assumes you drop the Plinko chip from Slot 5.

| $\frac{\# \text { of "right bounces" }}{}$ |  | $\frac{\text { winnings }}{6}$ |
| :--- | :--- | :--- |
| $0,4,8,12$ | $\longrightarrow$ | $\$ 10,000$ |
| $1,3,9,11$ | $\longrightarrow$ | $\$ 5000$ |
| 2,10 | $\longrightarrow$ | $\$ 100$ |
| 5,7 | $\longrightarrow$ | $\$ 0$ |

4. Repeat steps 1-3 five times. Record your results in the table below.

| Plinko Chip \# | \# of right bounces | amount you win |
| :---: | :--- | :--- |
| 1 |  | $\$$ |
| 2 |  | $\$$ |
| 3 |  | $\$$ |
| 4 |  | $\$$ |
| 5 |  | $\$$ |
| TOTAL WINNINGS: |  | $\$$ |

## Probabilities

1. Record the results of your five chips, along with those of your classmates, in the table below, where $x$ is the amount of your winnings from each Plinko chip

2. Change these frequencies into relative frequencies (probabilities for your experimental results)

| $x$ |
| :---: |
| Rel. Freq. |
| "probability" |

3. Compare your results to the "true" probability values for $x$

| $x$ | 0 | 100 | 500 | 1000 | 1000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $p(x)$ | $396 / 1024$ | $33 / 1024$ | $116 / 1024$ | $248 / 1024$ | $231 / 1024$ |

Why are the simulation results not identical to the "true" probability values?
4. Use your simulation data from before to calculate the expected winnings from one Plinko chip.
5. Use the table of "true" probability values to calculate the same expected value.
6. Now calculate the standard deviation for your simulation "probabilities" and for the "true" probabilities. Label them as $\sigma_{S}$ and $\sigma_{T}$ for Simulation and True results.

## Changing Starting Slots

What happens if you drop the chip from Slot 4? The table below gives you this information. Note, this is a different distribution than when we dropped the chip from Slot 5 .

| $\frac{\text { \# of "right bounces" }}{7}$ |  | $\frac{\text { winnings }}{}$ |
| :--- | :--- | :--- |
| $1,5,9$ | $\longrightarrow$ | $\$ 10,000$ |
| $2,4,10,12$ | $\longrightarrow$ | $\$ 5000$ |
| 3,11 | $\longrightarrow$ | $\$ 100$ |
| $0,6,8$ | $\longrightarrow$ | $\$ 0$ |

The table below indicates the probability distribution for Plinko if you drop a chip from Slot 4 rather than Slot 5.

| $x$ | 0 | 100 | 500 | 1000 | 1000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $p(x)$ | $355 / 1024$ | $58 / 1024$ | $157 / 1024$ | $256 / 1024$ | $198 / 1024$ |

1. Calculate the expected value of this probability distribution.
2. Calculate the standard deviation.
3. Which slot wins you more money in the long run: Slot 4 or Slot 5? Explain.
4. Which slot yield less variability: Slot 4 or Slot 5? Explain.
