Effective Use of Simulations in Teaching Probability and Statistics

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What Our Students Are Familiar With . . .

Note, most probabilities associated with these examples rely on the number of equally likely outcomes.

Probability As Limiting Relative Frequency

Definition (Probability of an Event, E)

The **probability of an event** E (recall that an event is a subset of the sample space of all possible outcomes), denoted $P(E)$, is the value approached by the relative frequency of occurrence of E in a very long series of replications (repeats of experiments with the same conditions) of a chance experiment, i.e.

$$P(E) = \lim_{N\to\infty} \frac{\text{number of times } E \text{ occurs}}{\text{number of replications,} N}.$$

Thus, if the number of replications, N, is quite large,

$$P(E) \approx \frac{\text{number of times } E \text{ occurs}}{\text{number of replications, } N}$$

Random Cell Phones

Suppose four identical phones are left on a table. One belongs to Heather, one to Craig, one to David, and one to Sam. You distribute a phone to each person. What is the probability that at least one person gets the right phone?

Image from

http://ww1.prweb.com/prfiles/2006/07/27/0000417675/ExampleofCellPhones.jpg

PLINKO!

The slots are, from left to right; $100, $500, $1000, $0, $10000, $0, $1000, $500, $100.

Image from

http://en.wikipedia.org/wiki/Image:Plinkoseason37.jpg

The popular Price is Right game show has a lot of games that are useful in teaching probability. Plinko! is one of those games. A contestant drops a chip from a slot at the top of the Plinko! board and it slides down, bouncing off of the pegs on the way, until it reaches the bottom. The contestant wins the amount of money displayed in the slot the chip lands in at the bottom.

Giving Credit Where Credit is Due ...

The following simulation was taken from a talk by

Dr. Matt Carlton and Mary Mortlock
of California Polytechnic State University, San Luis Obispo

entitled "Probability and Statistics Through Game Shows" given at the 2003 TCM Conference at NCSSM. The handout from the talk can be found at:
`http://courses.ncssm.edu/math/TCMConf/TCM2003/talks%202003/GAMESHOW1_carltonmort.pdf`

Simulating Plinko! with the TI-83

1. You can simulate one "drop" with the random integer function by using `randInt(0,1,12)[STO]->` L_1 on the TI-83. We let each 0 count as a left bounce and each 1 as a right bounce.
2. Tally and record the total number of right bounces. This can be recorded as `sum(L1)` from the home screen (note the `sum()` function is found under the `LIST -> MATH` menu, option number 5).
3. To find your location at the bottom of the Plinko board, use the conversion table below (which accounts for reflections off the wall). The table assumes you drop the Plinko chip from Slot 5.

# of "right bounces"		winnings
6	$\longrightarrow$	\$10,000
0, 4, 8, 12	$\longrightarrow$	\$1,000
1, 3, 9, 11	$\longrightarrow$	\$500
2, 10	$\longrightarrow$	\$100
5, 7	$\longrightarrow$	\$0

"So, what now?"

Besides having fun, what is the point of this simulation? There are several introductory probability topics that you can use this simulation to motivate:

- Approximating probabilities with relative frequencies with a large number of trials.
- Deriving the **expected value** formula.
- Tree diagrams.
- Binomial random variables (counting the number of "rights" in 12 bounces).

For additional useful teaching tools related to Plinko! check out the "Plinko! Probability from a TV Game" website:

http://mathdemos.gcsu.edu/mathdemos/plinko/

Sampling . . .

As it is rare to have access to an entire population, or to be able to get data on the entire population, we typically rely on extracting information about the population from information we gain from samples.

For example, if our population is the set of 300 songs on your MP3 player (in particular, the length of each song in seconds), we can take various random samples, creating a population of samples as it is. For each random sample of size n that is drawn we can concern ourselves with one or more of the following statistics:

- Average Song Length
- Maximum Song Length
- Difference between Max Song Length and Min Year Song Length
- etc.

Let's Sample . . .

Each person is to

1. Select three song lengths (chips) from the bucket,
2. Mark down the number of seconds associated with those three songs,
3. Return your chips to the bucket,
4. Determine the average length of your three selected songs.

Drawing Inferences . . .

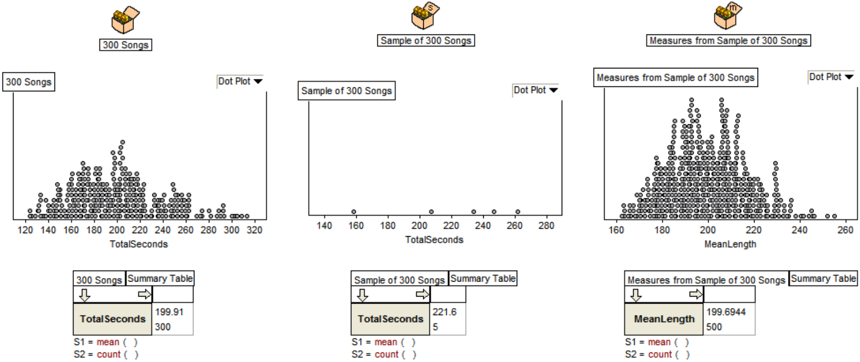
What we know:

- Multiple samples of a fixed size, $n = 3$.
- For each sample of size 3, we have average length of the songs in the sample, i.e. if $x =$ length of a song selected, then we've got a sample mean, $\bar{x}$.

Are you able to make any logical conclusions about the mean, standard deviation, or distribution of the 300 songs in the original population? Why or why not?

Using Fathom to Sample from a Population

Now suppose we did this for 500 samples with sample size $n = 5$.



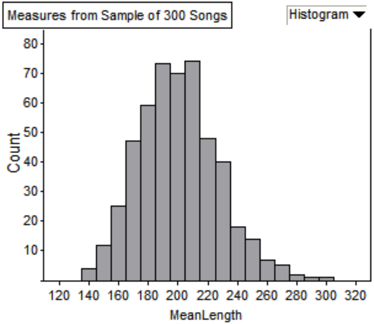
"So, what now?"

Besides the "Ooh" and "Aah" factor, what is the point of this simulation? Using Fathom to simulate sampling allows students to:

- Better understand the complicated concepts in sampling, and distributions of sample statistics.
- Derive the **Central Limit Theorem**.

Central Limit Theorem

Case with Sample Size $n = 2$

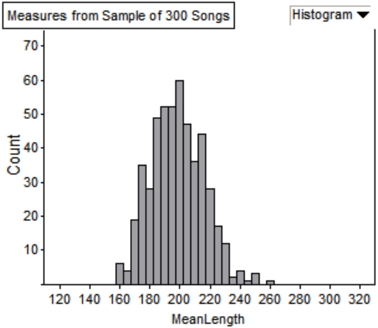


Measures from Sample of 300 Songs | Summary Table

MeanLength	200.283 500

S1 = mean ( )
S2 = count ( )

Case with Sample Size $n = 5$



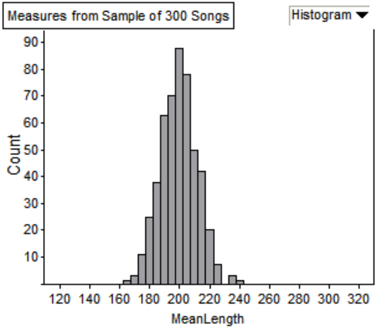
Measures from Sample of 300 Songs | Summary Table

MeanLength	198.42 500

S1 = mean ( )
S2 = count ( )

Central Limit Theorem

Case with Sample Size $n = 10$

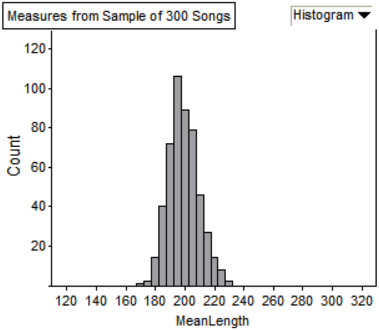


Measures from Sample of 300 Songs | Summary Table

MeanLength	199.5702
	500

S1 = mean ( )
S2 = count ( )

Case with Sample Size $n = 15$



Measures from Sample of 300 Songs | Summary Table

MeanLength	199.22173
	500

S1 = mean ( )
S2 = count ( )

References

- Carlton, Matt; and Mortlock, Mary. "Probability and Statistics Through Game Shows". Teaching Contemporary Mathematics Conference February 2003. http://courses.ncssm.edu/math/TCMConf/TCM2003/talks%202003/GAMESHOW1_carltonmort.pdf
- Lanier, Susie; and Barrs, Sharon. "Plinko! Probability from a TV Game." http://mathdemos.gcsu.edu/mathdemos/plinko/
- Rossman, Allan J; Chance, Beth L.; and Lock, Robin. *Workshop Statistics: Discovery with Data and Fathom*. Wiley, John & Sons, Incorporated. May 2001.