Populations in Stages with Leslie Matrices and MATLAB

1 New Zealand Sheep

The following table lists reproduction and survival rates for the female population of a certain species of domestic sheep in New Zealand, where sheep farming is a major segment of the economy. Sheep give birth only once a year, which dictates a natural time step of one year. In the species under consideration, sheep seldom if ever live longer than 12 years, which gives a natural stopping point for the age classes. [1]

Birth and Survival Rates for Female New Zealand Sheep
(from G. Caughley, "Parameters for Seasonally Breeding Populations",
Ecology, 48 (1967) pp 834 - 839)

Age (years)	Birth Rate	Survival Rate
0 - 1	0.000	0.845
1 - 2	0.045	0.975
2 - 3	0.391	0.965
3 - 4	0.472	0.950
4 - 5	0.484	0.926
5 - 6	0.546	0.895
6 - 7	0.543	0.850
7 - 8	0.502	0.786
8 - 9	0.468	0.691
9 - 10	0.459	0.561
10 - 11	0.433	0.370
11 - 12	0.421	0.000

Let x_1 represent the number of sheep in the 0-1 year age group; x_2 the number of sheep in the 1-2 year age group; x_3 the number of sheep in the 2 - 3 year age group; $\cdots$; and x_{12} be the number of sheep in the 11 - 12 year age group.

1. Draw the state diagram that models this situation. Make sure to label your components *neatly*.

2. Give the system of equations for $x_1(n+1)$, $x_2(n+1)$, $x_3(n+1)$, $\cdots$, $x_{12}(n+1)$ in terms of $x_1(n)$, $x_2(n)$, $x_3(n)$, $\cdots$, $x_{12}(n)$.

3. Rewrite your system of linear equations above as a matrix equation.

4. In MATLAB, define your Leslie matrix as L. (**Recall:** In order to define a variable or matrix in MATLAB, you must use the `=`, e.g `A =[1,2;3,4]` defines the matrix A to be the 2×2 matrix with entries 1,2, 3 and 4.).

5. (**Note:** A matrix with only one column is called a **vector**). Suppose that initially there 200 sheep in each age group. Define `X0` to be the vector of initial population values.

6. Using MATLAB and the power operator ($\wedge$), calculate the number of females in each age group and in total after fifty years.

7. Use MATLAB to determine of the number of females in each age group reaches a *steady state*.

[1] From "Leslie Growth Models, Part 1" at `http://www.math.duke.edu/educatin/ccp/materials/linalg/leslie/lesl1.html`

2 The Life Expectancy of Possums

The following table lists reproduction and survival rates for the female population of possum in the United States. Suppose possums give birth only once a year, which dictates a natural time step of one year. Due to natural life span and traffic possums seldom if ever live longer than 5 years, which gives a natural stopping point for the age classes.

Birth and Survival Rates for Female Possums

Age (years)	Birth Rate	Survival Rate
0 - 1	0.0	0.6
1 - 2	1.3	0.8
2 - 3	1.8	0.8
3 - 4	0.9	0.4
4 - 5	0.2	0.0

Let x_1 represent the number of possums in the 0-1 year age group; x_2 the number of possums in the 1-2 year age group; x_3 the number of possums in the 2 - 3 year age group; x_4 the number of possums in the 3 - 4 year age group; and x_5 be the number of possums in the 4 - 5 year age group.

1. Draw the state diagram that models this situation. Make sure to label your components *neatly*.
2. Give the system of equations for $x_1(n+1)$, $x_2(n+1)$, $x_3(n+1)$, $x_4(n+1)$, $x_5(n+1)$ in terms of $x_1(n)$, $x_2(n)$, $x_3(n)$, $x_4(n)$, $x_5(n)$.
3. Rewrite your system of linear equations above as a matrix equation.
4. In MATLAB, define your Leslie matrix as L.
5. Suppose that initially there 194 possum in the 0 - 1 age group, 82 in the 1 - 2 age group, 55 in the 2 - 3 age group, 22 in the 3 - 4 age group, and 6 in the 4 - 5 age group. Define `X0` to be the vector of initial population values.
6. Using MATLAB, calculate the number of female possums in each age group and in total for the years indicated below and fill those values in the appropriate columns of the chart. Round your answer to the ONE DECIMAL PLACE.

Age Group	Initial Year	After 1 Year	After 2 Years		After 11 Years	After 12 Years	After 13 Years	After 14 Years
x_1 (0 - 1)	194							
x_2 (1 - 2)	82							
x_3 (2 - 3)	55							
x_4 (3 - 4)	22							
x_5 (4 - 5)	6							
Total Female Population	359							

7. Now calculate the rate at which the population is changing over each of those years by filling in the table below. Round your answers to THREE DECIMAL PLACES.

	Between Years 0 & 1	Between Years 1 & 2		Between Years 11 & 12	Between Years 12 & 13	Between Years 13 & 14
$k = \dfrac{\text{Pop. at Year } n+1}{\text{Pop. at Year } n}$						
Total Change: $k-1$						

8. Use the command `eig(L)` to find the eigenvalues of the Leslie matrix. List those eigenvalues below:

9. Which eigenvalue is the dominant eigenvalue? State how you know this.

10. What does the value of the dominant eigenvalue have to do with your answer to question 7? EXPLAIN.