

Some Excel Basics for Mathematical Modeling

As with learning most new concepts, a particular example is very helpful. Here we'll rely on a typical exponential growth example where rabbits reproduce at a fixed rate per year (say 10% per year). Here n indicates the number of years that have passed since you started studying the rabbit population and $u(n)$ is the number of rabbits present after n years.

Example 1 (Rabbit Example) *Remembering Present = Past + Change, we can determine the difference equation the difference equation that models the rabbit population*

$$\underbrace{u(n)}_{\text{Present}} = \underbrace{u(n-1)}_{\text{Past}} + \underbrace{0.10 * u(n-1)}_{\text{change}}$$

We'll also need an initial value, the number of rabbits in our population when we started our study. Let's suppose we started with 50 rabbits, then we know

$$u(0) = 50$$

Suppose you want to calculate several values of $u(n)$ and graph them using an Excel spreadsheet.

Calculating Values of $u(n)$

1. Put in column headings for n and $u(n)$ at the top of the two columns you're going to use. (We'll assume these are typed in cells A1 and B1 here.)
2. Enumerate the values of n you're going to use, making sure that you START WITH YOUR INITIAL VALUE OF n , here it's 0.

To make this easier than typing all of the numbers out, you can create a formula for increasing numbers and "drag" this formula down to the lower cells. To do this:

- (a) Type in your initial value of n in the first cell below your heading. (We'll assume this is typed in cell A2 here.)
- (b) In the next cell down, type

$$= [\text{Cell Address for Initial } n] + [\text{Step Size}]$$

In general your step size is 1, indicating you're adding 1 to the previous value of n to get your next n value. Rather than typing in the Cell Address for your Initial n value, you can simply "click" once on this cell with your cursor and the cell address will be inserted in your formula.

For this particular example you'd type the following in cell A3:

$$= A2 + 1$$

- (c) Copy this formula down to the cells below by letting your cursor hover over the bottom right corner of cell A3 until it becomes a "+", then holding down your left mouse button (on a PC) drag the cursor down to the cells you want to copy this formula into. When you're over the last cell you want to fill, release the mouse button. Note, in doing this, Excel increments the cell address referenced to always be the cell directly above the cell the formula is typed in. This is called *relative addressing* in Excel.
3. Input your initial value of u in the first cell under your $u(n)$ heading. (Here, we'd type in a 0 in cell B2).

4. Calculate the subsequent values of $u(n)$ using a similar “formula drag” as you did for filling in the n values.
 - (a) In the cell just below the initial u value, type the formula (thus you must start with an “=”) for the right hand side of the difference equation, but everywhere you see a $u(n - 1)$ replace this with the address of the u value in the cell directly above (recall, you can do this with a mouse click if you prefer).

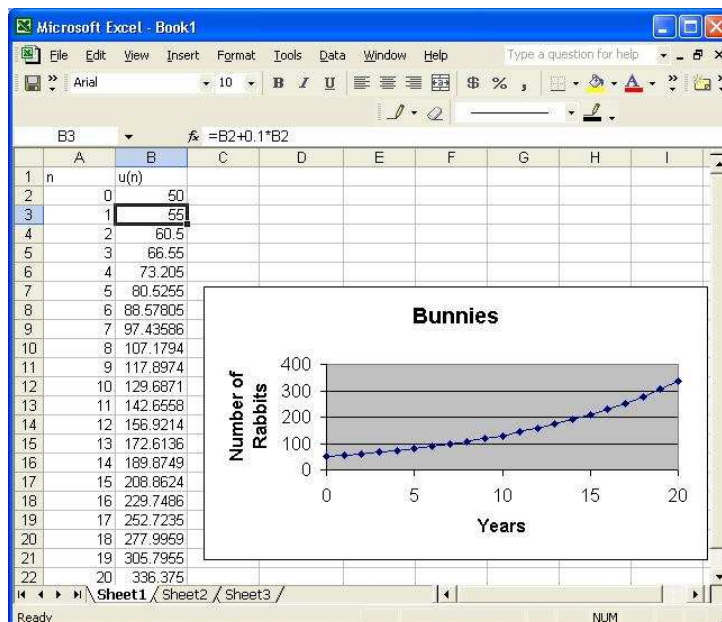
For this particular example, you’d type the following in cell B3:

$$= B2 + 0.10*B2$$

- (b) Now “drag” this formula down to the subsequent cells, by hovering the cursor over the lower right corner of the cell until it becomes a “+”, holding down the left mouse button, and dragging the cursor to the lowest cell you want to fill in and releasing the mouse button.

Plotting Values of $u(n)$

1. Go to a clear portion of your Excel spreadsheet and click your cursor on an empty cell.
2. Go to the **Insert** menu at the top and select **Chart**. This should bring up a pop-up box that will walk you through the creation of your chart.
3. Select the **XY (Scatter)** plot type, and then the **Scatter Plot with values connected by smooth lines** option, then click the **Next** button at the bottom of your pop-up box.
4. Under the **Data Range** tab, click in the **Data Range** box, then go back to your spreadsheet and select the two columns of numbers you have for n and $u(n)$, but don’t include the actual name headers (note: to select the columns, simply go to one corner of your columns, hold down the left cursor and then move your cursor to the opposite corner of the data, then release your cursor). Click the **Next** button when this is done.
5. Fill in any titles for the graph or axes, then click **Next**.
6. Lastly, select the option to place your chart as an object in the sheet.



Parameterizing Your Equations

Often times it is of interest to see what happens if we change the value of some of our constants (parameters) in a problem. In the rabbit example, we have two parameter values: the growth rate and the initial number of rabbits in the population. Although it is easy to change the initial number of rabbits, simply type over the value in the A2 cell and hit enter. Adjustments are automatically made to the following population cells because of the sequential cell referencing. In order to change the growth rate in the original problem, we must change it physically in each cell formula (or retype the original and re-“drag”). This can be made easier by having one cell on the spreadsheet hold the value of the parameter, and referencing this cell in the formulas.

1. In cell D1 type a name for the parameter, such as Birth Rate.
2. Next to this, in cell E1 type the decimal value of the parameter, 0.10, for our original example.
3. Now readjust the formula for the rabbit population, starting with cell B3. Rather than having “0.10*B2”, we’ll use the cell address to replace our 0.10 parameter. NOTE: in order to “freeze” this cell reference, so when we drag our formula down we don’t unintentionally increment the reference to the parameter cell, we must put dollar signs in front of the column and row heading. This is called *absolute addressing*.

For this particular example, you’d type the following in cell B3:

$$= B2 + \$E\$1*B2$$

4. Now “drag” this formula down to the subsequent cells, by hovering the cursor over the lower right corner of the cell until it becomes a “+”, holding down the left mouse button, and dragging the cursor to the lowest cell you want to fill in and releasing the mouse button.
5. In order to change the value of the birth rate in the problem, one only needs to adjust the value stored in the parameter cell, E1, now and hit enter.

Naming a Parameter Cell

There is still one more option when using parameters stored in a special place on the spread sheet. You can name a cell, overwriting the default Column Letter - Row Number addressing in Excel. For example, rather than referring to the parameter cell as E1, we can name it **BirthRate** (no spaces allowed in the names). To do this simply:

- Click on the parameter cell. To the lower left portion of the tool bar, you will notice a white space with E1 showing. Click on this space, delete the E1 and type **BirthRate** in it’s place. Then hit enter. Now each time you click on that cell, it’s address will show as **BirthRate**. Simply change your formulas to reference **BirthRate** rather than **\$\$\$1**.

OR

- 1. Click on the parameter cell.
 2. Go to the menu option **Insert** → **Name** → **Define**.
 3. If you have a label typed next to your parameter cell, Excel will automatically suggest this name for your cell name, or you can overwrite any of its suggestion.
 4. Make sure that the cell being referenced is the one you intended, and hit Enter.

Naming a cell can be helpful in teaching as it indicates more explicitly the meaning of each parameter in an equation, rather than a simple cell location.

Using Slider Bars

Rather than typing in different values for parameters and noting the change on the solution graph, you can “speed up” this process by adding a slider bar to your spreadsheet that will allow students to adjust parameter values on the fly. NOTE: The scroll bar values can be integer values only, so when we want to use them to control something with decimal values (such as the birth rate), we must make an adjustment.

1. First get the Forms toolbar by going to the menu bar and select **View** → **Toolbars** → **Forms**.
2. Click on the Scroll Bar icon and place it on your spreadsheet by clicking and HOLDING DOWN the left mouse button and dragging diagonally until you have the outline of the box you want for your slider bar. Then release the mouse button.
3. Make sure the slider bar has been selected (the resize corners will be visible).
4. Double click on the inside of the scroll bar to get a pop-up box that allows you to define the settings for the scroll bar.
 - Incremental change is the amount by which the scroll bar will change when the user clicks on one of the arrows at the end of the bar.
 - Page change is the amount by which the scroll bar will change when the user clicks inside the scroll bar to the left or right of the bar.
 - Cell link lists the address of the cell whose value is to be controlled by the scroll bar.
5. For this example we will let the scroll bar take on values 0, 1, 2, ..., 100 and we'll let our birth rate be $1/100^{th}$ of the scroll bar value. So define minimum value to be 0, maximum value to be 100, incremental change to be 1, page change to be 10 and let the cell link be E5.
6. Finally, we will change our parameter cell formula to be a function of the scroll bar value.
For this particular example, you'd type the following in cell E1:

$$= 0.01 * E5$$

7. Now toggle the scroll bar and watch the effects on the graph.
8. If you want to change scroll bar settings, activate the scroll bar by right clicking on top of it, then double click on the center of the scroll bar.

