

Mathematical Models of Population Growth: Communicable Disease Data¹

How to “Play”

Each student must be given a unique three-digit ID number. The simulation is split into five stages, with each stage lasting two minutes. At the start of a new stage, students are to get up from their seat and walk around the room, having one-on-one encounters with other students. For each encounter they are to shake hands, exchange ID numbers (recording the other person’s ID number on their sheet), roll their die, and record the sum of the roll on their data sheet. When time has run out for that stage, students are to return to their original seat and wait for the start of the next stage. Once the stages have been completed, announce the sums of die that indicate “risky” encounters (e.g. if sum of die is greater than 7), and ask students to mark all risky encounters on their sheet. Next pick an ID number at random to indicate the infected student at the start of the simulation. Identify which students in the first stage had a risky encounter with this infected individual, and add their numbers to a list of infected people. Next, identify the students who had risky interactions with one of the students on the infected list. They will become infected in this stage. Continue this process until all stages have been covered.

Ways This Experiment Could Apply to Different Math Classes:

Algebra 1

- This activity provides an opportunity to practice plotting data points.
- This activity also reinforces vital skills in reading and interpreting graphs.
- Early simulation behavior provides an opportunity for discussion of exponential models and their graphs (and why, it is not the appropriate model in terms of long term approximation).

Pre-Calculus

- This simulation provides an excellent example of a logistic data, and it’s initial similarity to exponential data.
- For classes that discuss the logistic function, the TI-83’s logistic regression capabilities can be used to determine a logistic function that best fits the data.
- Students can use data and/or the logistic regression equation to calculate average rates of change.
- Discuss the concavity of a graph and it’s graphical and physical interpretation, in terms of the spread of the disease.

Calculus

- Discussion of average rates of change can be extended to instantaneous rates of change (of the continuous case) and their interpretation with respect to the disease simulation.
- Extend the rates of change discussion to include discussions about concavity, the second derivative as the rate of change of the rate of change, the point of inflection, and their interpretation.
- Students can explore limiting values in terms of the interpretation in the simulation, in terms of the curve, and in terms of the logistic function $\left(L(x) = \frac{C}{1+Ae^{-Bx}}\right)$.

Statistics

- Students could explore the distribution of the sum of the roll of two die and the probability that a random encounter is “risky”.
- This simulation provides an excellent source of time series data, and promotes the discussion of randomness.

¹This activity was adapted from an article by Marilyn Stor and William L. Briggs, “Dice and Disease in the Classroom”, *Mathematics Teacher*, September 1998, Volume 91, Issue 6, pp. 464 - 468.