Research Statement
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Historians have often characterized British mathematics of the early nineteenth century as depressed and insular. Several British mathematicians of the period also shared this evaluation. The strongest expression of this view came from Charles Babbage (1791-1871), who devoted his entire 1830 work, *Reflections on the Decline of Science in England and on Some of Its Causes*, to this theme. In his "Introductory Remarks," Babbage stated that

> [i]t cannot have escaped the attention of those, whose acquirements enable them to judge, and who have had opportunities of examining the state of science in other countries, that in England, particularly with respect to the more difficult and abstract sciences, we are much below other nations, not merely of equal rank, but below several even of inferior power.[1]

To strengthen his claim, Babbage quoted John Frederick William Herschel (1792-1871), who declared that in Britain, "whole branches of continental discovery are unstudied, and indeed almost unknown, even by name. It is in vain to conceal the melancholy truth. We are fast dropping behind. In mathematics we have long since drawn the rein, and given over a hopeless race."[10] However, by 1870, the French geometer Michel Chasles (1793-1880) painted for his countrymen a completely different picture of the mathematical situation in Britain:

> . . . a mathematical society was founded in London in 1865 with a membership of one hundred, and this number is increasing; a society whose *Proceedings*, like those of the Royal Society of London,. . . publishes abstracts, more or less extended, of many papers. Is not [the existence of the *Proceedings of the London Mathematical Society*], which we applaud, an ingredient of future superiority in mathematical culture that should worry us? [4]

What happened to British mathematics in the space of only 40 years to produce these two profoundly different perceptions? To what extent do these contemporary perceptions hold true? Which factors enabled Britain, a country not quite at the mathematical forefront during the nineteenth century, to give us the first national mathematical society in Europe or America (the London Mathematical Society) as well as the pure and applied mathematical powerhouses of Arthur Cayley (1821-1895), J.J. Sylvester (1814-1897), William Rowan Hamilton(1805-1865), Lord Kelvin (1824-1907), and G.G. Stokes (1819-1903)?

In my efforts to address these questions, I investigate the structure, support, and content of mathematics in scientific journals in order to trace more accurately the development of mathematics and mathematicians in nineteenth-century Britain. Viewing scientific papers contained in scientific journals as significant indicators of research, I consider the scientists who authored, read, and responded to papers in a given area within a given group of journals as a publication community. My research examines the nineteenth-century British mathematical publication community from three different yet complementary points of view.

It focuses on the structure of journals, isolating the factors that affected journal foundation, organization, operation, and specialization with respect to mathematics. It focuses on contributors both domestic and foreign, providing a profile that highlights mathematical training, locates mathematical centers, and measures the extent of professionalization and internationalization in this publication community. Finally, it focuses on the mathematical contributions themselves, revealing the subjects that interested and engaged this group of contributors and placing these subjects within national and international contexts. Through these three foci, my research provides a balanced investigation of the structures, members, and products of the British mathematical publication community throughout the nineteenth century.

Why does the nineteenth century define the time period for this study? During that century, many of the structures and standards for mathematical communication were in a state of introduction, development and evolution. One process we take for granted today, the refereeing of mathematical articles, was, during the nineteenth century, only slowly beginning to receive acceptance and use. For instance, the Royal Society of London only began requiring written referee reports in 1832, and as late as 1898 was it able to ensure the anonymity of its referees. By reading the manuscripts of many of the editors, contributors, and publishers of independent mathematical journals, I have uncovered an intriguing web of far-reaching visions, competing personalities, and economic worries that accompanied the development of this journalistic process. The referees' reports of several of the mathematical Fellows of the Royal Society have also provided first-hand accounts of mathematicians upholding the quality and defining the place of mathematics in general science society journals through refereeing. By defining what was publishable, mathematical editors and referees established limits on the style and depth of mathematics published in their journals; they built the stage on which British mathematics was presented to the world. My study — through the various correspondence collections I have uncovered— provides a never-before-seen glimpse into how refereeing, so essential to mathematics today, became institutionalized.

Given that the nineteenth century represents an exciting, dynamic period in the development of mathematics, why focus the study of this development on nineteenth-century *Britain*? While France and Germany exchanged places as the number one and number two nineteenth-century mathematical powers, Britain remained at number three. However, during the era of Isaac Newton, Britain was recognized as a scientific leader. Studying Britain during the nineteenth century, then, allows us to investigate why this country lost its leading position in mathematics and why this position was so hard to regain. Studies exist on a number of individual British mathematicians, areas of mathematics, and institutions for mathematical training, but there has never been a comprehensive study of the nineteenth-century British mathematical community as a whole. Such a study can give a more complete picture of factors behind Britain's efforts to regain its place on the international mathematical stage in the nineteenth century.

While my research represents the first comprehensive study of nineteenth-century British mathematics, the analytical tool that I use to identify this community also represents an innovative departure from existing studies of other national mathematical communities. Gispert (see [8]) used the membership of the Société mathématique de France to identify a community of French mathematicians, their mathematical production, and the institutional changes in which they were involved. Biermann (see [2]) used the University of Berlin as the focus in his analysis of educational and professional development of German mathematicians. Similarly, Parshall and Rowe (see [11]) traced the emergence of the American mathematical research community through the formation of mathematical schools in American universities and the specific initiatives of some of their leaders, including J.J. Sylvester at the Johns Hopkins University and E.H. Moore (1862-1932) at the University of Chicago. Instead of using universities or societies to define the community that I study, I have identified the members of this community through the mathematical contents of scientific journals. However, once I identified this community, my research, like the existing studies cited above, investigates the educational and professional developments surrounding this group of mathematicians and so can fruitfully be used in further comparative studies among national mathematical communities.

After determining the domestic members of the British publication community, I employ collective biography, or prosopography, to understand better important community factors such as mathematical training and professional opportunities. Systematically reviewing the nineteenth-century contents of nine British scientific journals has yielded over 500 domestic contributors. I have searched university alumni lists and society membership rosters as well as several national and institutional biographies in order to cull information about this group's education, society involvement, and career paths. This research has confirmed earlier studies that have located Cambridge as the primary mathematical center of nineteenth-century Britain; well over half of these domestic contributors were trained or worked in this institution. Cambridge owed much of its preeminence in mathematics to its mathematical Tripos examination. This examination, for over half the nineteenth century, was a mandatory hurdle for the BA degree at Cambridge (even for those students interested in non-mathematical areas). While it encouraged students to study mathematics, its contents also tightly circumscribed the mathematical subject areas that students were motivated to study. Furthermore, attaining high marks on this examination formed one of the first mathematical goals for many of the members of the British mathematical publication community: over half of the domestic contributors identified in this study scored as wranglers (the highest honors bracket) on the Tripos. Thus, the Tripos as well as other facets of the Cambridge educational system, shaped the mathematical goals of a substantial proportion of young British mathematicians. What happened to a Cambridge student after the examination? Unlike their French and German counterparts, these students had no organized program of postgraduate research to follow. However, the last quarter of the nineteenth century witnessed the adoption of dissertations for the adjudication of university fellowships and prizes; with these innovations, the research possibilities for graduates became better defined.

While providing a richer picture of the educational paths of the domestic members of the British mathematical publication community, my analysis has also illuminated their career paths. After obtaining their mathematical training, the majority of these mathematicians stayed in academia as university fellows, lecturers, administrators, and professors. A considerable number also found employment as school teachers or private tutors. However, British mathematicians also worked in the non-academic fields of law, the church, and business. In the nineteenth century, the career path of a British mathematician was not nearly as clear-cut as it is today. For many of these researchers including those in academia, mathematics was an after-hours pursuit.

While some of the educational traditions and career limitations in nineteenth-century Britain could stifle mathematical growth, mathematicians actively worked to improve the situation of British mathematics. One initiative, the internationalization of British mathematics, was enthusiastically encouraged and embraced by a small but powerful group of mathematicians. In order to bring Britain to the international arena, this group lobbied for society medals and memberships for mathematicians from abroad, translated important international articles into English, and actively encouraged foreigners to participate in their journals. In [7], I examined some of the fruits of this labor by focusing on the international members of the British mathematical publication community. Through an exhaustive review of 20 British scientific journals, I discovered a slow but steady growth in international participation from a variety of countries with established, as well as emerging, mathematical communites. The activities of one international member of the British mathematical publication community, Charles Hermite (1822-1901), included publishing in several British scientific society and independent mathematical journals as well as serving on the editorial staff of the *Quarterly Journal of Pure and Applied Mathematics* from its founding in 1855. In his most notable contribution to the *Quarterly Journal*'s predecessor, the *Cambridge and Dublin Mathematical Journal* [9], Hermite débuted his law of reciprocity in invariant theory. A function of the coefficients of a binary form $Q(x, y)$ that changes under a linear transformation only up to a power of the determinant is called an invariant, while a function of the coefficients and variables of Q with the same property is called a covariant (thus, an invariant is just a special kind of covariant). Hermite's law of reciprocity states that the number of covariants of the p^{th} order in the coefficients of a binary form of degree m is equal to the number of covariants of the m^{th} order in the coefficients of a binary form of degree p. In the early 1850s, invariant theory was focused on systematically determining the covariants and invariants for binary forms of each successive degree. Thus, Hermite's surprising result, presented in a British mathematical journal, effectively cut the work of invariant theorists in half by doubling the number of known invariants and covariants. This kind of participation in British mathematical journals added legitimacy to Britain's efforts at building a mathematical community.

British mathematicians also gained legitimization for their community-building efforts by presenting their work in international publication venues. I extended my study of internationalization, by investigating the reciprocal group of nineteenth-century British mathematicians who published abroad. By looking at the mathematical training, employment, awards and society participation of these mathematicians, I discovered a concentrated, influential group of British mathematicians with international goals firmly in focus. These reciprocal studies of international initiatives refute earlier perceptions of nineteenth-century mathematics in Britain as insular and isolated.

Using the journal articles of nineteenth-century British mathematicians to identify a publication community, active mathematicians regardless of their profession or educational background are included in the sample space. This is especially critical for understanding the nineteenth century, a period in which the educational and professional structures for British mathematics were in a state of profound development. Other inclusionary criteria, based solely on society memberships, university ties, or career paths, would allow mathematicians vital to our understanding of nineteenth-century British mathematics to fall through the cracks. My study's focus on the journal articles also allows me to view nineteenth-century mathematical developments in their initial form of presentation. Looking at the contents of the journal articles themselves allows me to highlight memorable, forgettable, and forgotten contributions, reveal topics that contemporary mathematicians viewed as popular or important, explore the theorems and theories produced by British mathematicians, and indicate trends of mathematical research in nineteenth-century Britain.

The birth of one very active line of nineteenth-century British research, invariant theory, is recorded on the pages of the *Cambridge Mathematical Journal*. George Boole (1815-1864), a self-taught teacher in Lincoln who acquired university ties only later in life, introduced British mathematicians to the idea of invariance in 1841 [3]. Given a binary quadratic form $Q(x, y) = ax^2 + 2bxy + cy^2$, Boole used the linear transformation:

$$\begin{gathered} x \to mx' + ny' \\ y \to m'x' + n'y' \end{gathered}$$

where $mn' - m'n \neq 0$, and m, m', n, and n' are real, to transform $Q(x, y)$ into a new binary quadratic form $R(x', y') = \alpha x'^2 + 2\beta x'y' + \gamma y'^2$ by substituting the transformed variables x and y into Q. Here,

$$\begin{gathered} \alpha = am^2 + 2bmm' + cm'^2, \\ \beta = amn + bmn' + bm'n + cm'n', \text{ and} \\ \gamma = an^2 + 2bnn' + cn'^2. \end{gathered}$$

Boole showed, that

$$\beta^2 - \alpha\gamma = (mn' - m'n)^2(b^2 - ac),$$

that is, the discriminant of $Q(x, y)$ equaled that of $R(x', y')$ up to a factor of the square of the determinant of the linear transformation. Thus, the discriminant represented the simplest example of an invariant. Boole extended his findings to binary cubic forms, and presented a method for finding invariants that employed the elimination of the variables of these forms through partial differentiation. Through the *Cambridge Mathematical Journal*, Boole introduced British mathematicians to ideas that some, most notably Cayley and Sylvester, would develop into a new field of research and actively pursue throughout the rest of the nineteenth century.

While my research gives new and unique insights into the development of nineteenth-century British mathematics, it opens up many possibilities for future investigations. In particular, my current research on the mathematical products presented in the journals identifies trends of mathematical research in nineteenth-century Britain to which I will give more detailed consideration in the future. One possible study would investigate nineteenth-century geometrical research in Britain, comparing it to contemporary geometrical developments in France and Germany. I would also like to take advantage of the capability of my current research to uncover the overlooked but important mathematicians obscured by the "great men of mathematics." While understanding the life and work of the most renowned mathematicians is important, if we allow their shadows to hide lesser known mathematicians, we impoverish our perception of the mathematical developments surrounding the mathematical giants. For example, with this higher resolution, I should be able to provide valuable insights into the current understanding of the formation and development of a British school of invariant theory (see [5], [6], and [12]). While some believed that Hilbert's work on invariant theory in the 1890s sounded its death knell, the area has repeatedly reappeared in many different guises. Thus, a better grasp of the historical developments of nineteenth-century invariant theory will help inform our understanding of the various reincarnations of the subject through the twentieth century and today. Future studies on mathematical trends and schools will also allow me to say much more on developments in British mathematics through the century, and their ties to the international mathematical community. These and other future avenues of study that my dissertation research has suggested give me direction and purpose for future research, which I am eager to pursue.

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